UNIT-①

Introduction - conduction heat transfer

* modes of heat transfer

→ In general there are three modes of heat transfer. they are as follows...

① conduction ② convection ③ Radiation

① conduction :-
When the heat is transferred from a higher temperature region or body to a lower temperature region, only when both the bodies are in direct physical contact (solid, liquid, Gas). It is called conduction.

Example :- When two metallic or nonmetallic bodies are in direct contact then the heat flows from warm body to a cold body, until both the bodies attain same temperature (i.e equilibrium temperature)

② convection :-
heat is transferred from one body to other, only in the presence of fluid (air or liquid) Generally, they are 2 types convection

(i) natural convection (ii) Forced convection

Example :- cooling of IC engine by forced circulation of water of air.

③ Radiation :- In this mode, heat is transferred in the form of radiant energy or wave motion from higher temperature region to lower temperature region. Thus heat is transferred even in the absence of a medium.

Example:-

① heat transfer from sun to earth

② Transfer of heat from a thermal flask.

Temperature field and Temperature Gradient

* Temperature is a function of space coordinate and time

case-①:- It is time independent

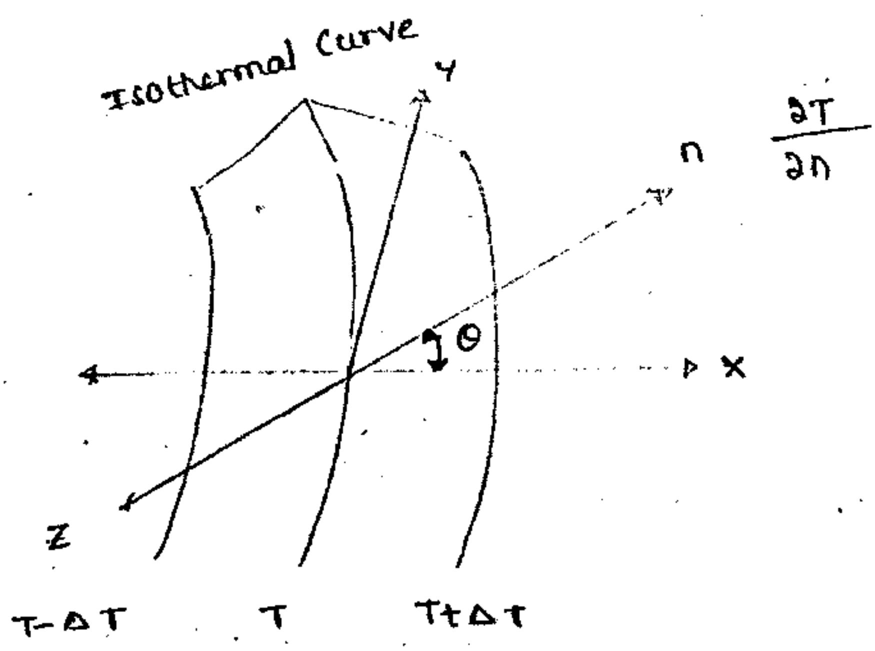
$$T = f(x,y,z) \quad \left[\frac{\partial T}{\partial t} = 0\right] \quad \text{(steady state)}$$

case-②:- It depends on two space coordinates and time dependent

$$T = f(x,y,t) \quad \left[\frac{\partial T}{\partial z} = 0\right]$$

If it is steady

$$T = f(x,y) \quad \frac{\partial T}{\partial z} = 0, \quad \frac{\partial T}{\partial t} = 0$$



By this we can calculate Temperature T along space coordinates.

$l, m, n \longrightarrow$ direction cosines

$$\frac{\partial T}{\partial x} = l \frac{\partial T}{\partial n}$$

$$\frac{\partial T}{\partial y} = m \frac{\partial T}{\partial n}$$

$$\frac{\partial T}{\partial z} = n \frac{\partial T}{\partial n}$$

Conduction: Fourier law

→ The rate of heat transfer is directly propotional to the area normal to the direction of the heat flow and the temperature gradient in that direction.

$$Q \propto A \times \frac{\partial T}{\partial n}$$

$$Q = -kA \frac{\partial T}{\partial n} \qquad [-k \text{ is reduction in heat}]$$

where Q = heat transfer rate, k = thermal conductivity of material

$$q = \frac{Q}{A} = -k \frac{\partial T}{\partial n}$$

here q = heat flux

$$k = k_e + k_L \qquad k_L = \text{lattice}$$

① A stainless steel plate 2cm thick is maintained at a temperature 550°c at one place and 50°c on the other. the thermal conductivity of stainless steel at 300°c is 19.1 W/m-K. compute the heat transfer through the material per unit area

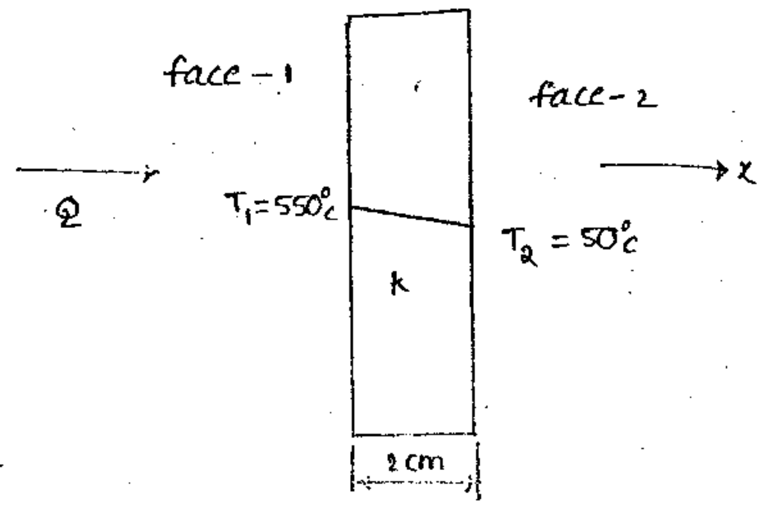
Sol:- Given

Stain less steel plate thick $t = 2\,cm$

Temperature at one end $T_1 = 550°c$, Temperature at other end $T_2 = 50°c$, thermal conductivity $k = 19.1\ W/mk$

W.K.T Fourier law

$$Q = kA \frac{\partial T}{\partial n}$$



$$q = \frac{Q}{A} = \frac{kA \frac{\partial T}{\partial n}}{A}$$

$$= k \times \frac{\partial T}{\partial n}$$

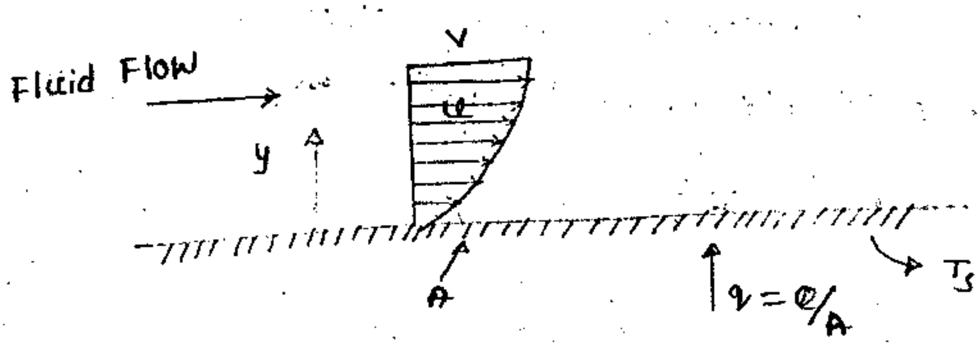
$$q = 19.1 \frac{W}{mk} \times \frac{dT}{dx} = 19.1 \frac{[(550+273) - (50+273)]}{t}$$

$$q = 19.1 \frac{W}{mk} \frac{[823 - 323]\,k}{2 \times 10^{-2} m}$$

$$q = 477.7 \times 10^3 \frac{W}{m^2}$$

Convection:- Newton's law of cooling

It is defined as Rate of heat transfer is directly proportional to the Area of surface and temperature difference between the surfaces



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$$Q \propto A \Delta T$$

$$Q = hA\left[T_s - T_\infty\right] \longrightarrow$$ (Newton law of cooling)

$$q = \frac{Q}{A} = h\left[T_s - T_\infty\right]$$

$$Q = -kA\left(\frac{\partial T}{\partial n}\right) = hA\left(T_s - T_\infty\right)$$

$$h = \frac{-k}{(T_s - T_\infty)}\left[\frac{\partial T}{\partial n}\right]$$ where h = convective heat transfer coefficient

① If a Flat plate of length 1m and width 0.5m is placed in an air stream at 30°c blowing parallel to heat. The convective heat transfer coefficient is 30 W/m^2K. calculate the heat transfer if the plate is maintain at the temperature of 300°c

Sol:- given that

Flat plate length = 1m, width = 0.5m, convective heat transfer coefficient $h = 30\ W/m^2K$, plate maintain temperature

$T_s = 300°c$, $T_\infty = 30°c$

w.k.T $Q = hA\left[T_s - T_\infty\right]$

$$Q = 30\ \frac{W}{m^2K} \times (1 \times 0.5)m^2\left[(300+273) - (30+273)\right]$$

$$= 30 \times 0.5\ \left[573 - 303\right]$$

$$Q = 4050\,W = 4.05\ kW$$

Thermal radiation :- stephen Boltzman law

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The rate of heat transfer is directly propotional to the fourth power the absolute temperature and surface area

→ Energy emitted by the radiation is propotional to fourth power of Absolute temperature

$$Q \propto A T^4$$

$$Q = \sigma A T^4$$

Where σ = stephen Boltzman constant $= 5.697 \times 10^{8}\ W/m^2 K^4$

(i) Two Black Bodies

$$Q = \sigma A_1 (T_1^4 - T_2^4)$$

(ii) Between Black Body and Grey bodies

$$Q = \sigma \varepsilon A_1 [T_1^4 - T_2^4]$$

Where ε → Emissivity factor

$$Q = \sigma \varepsilon A_1 F (T_1^4 - T_2^4)$$

F → view factor or shape factor → Geometry of Body

① A radiator in a domestic heating system operates at a surface temperature of 55°c Determine the rate at which the emits radiant heat per unit area if it behaves like a black body

Sol: given that

temperature $T_1 = 55^\circ c$

W.K.T $Q = \sigma \times A_1 [T_1^4 - T_2^4] \Rightarrow q = \frac{Q}{A} = \sigma \times [T_1^4 - T_2^4]$

$q = 5.697 \times 10^{-8} \times [273+55]^4$

$q = 656.22 \ W/m^2$

$q = 0.656 \ kW/m^2$

Electrical Analogy

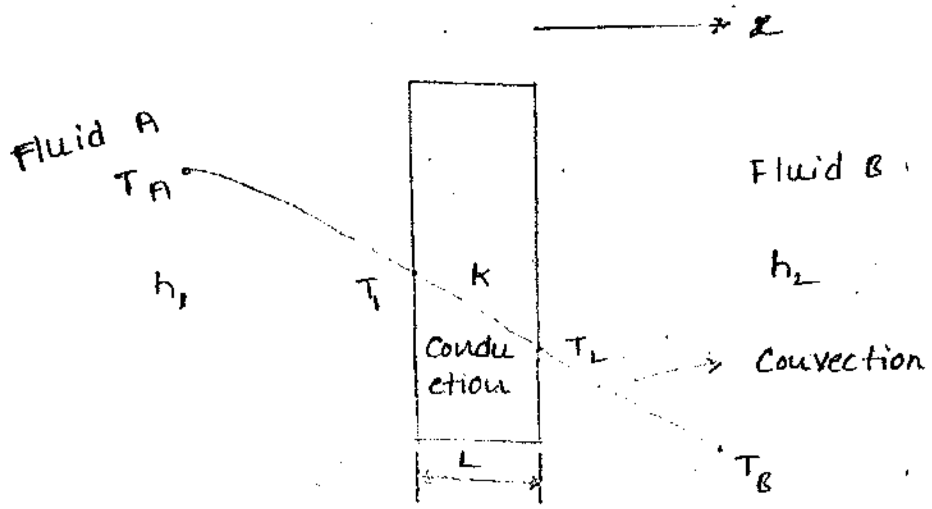


Fig (a)

conduction :-

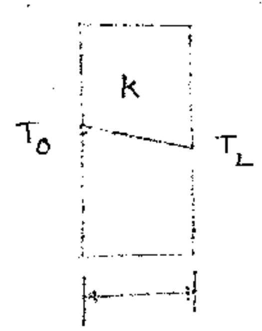
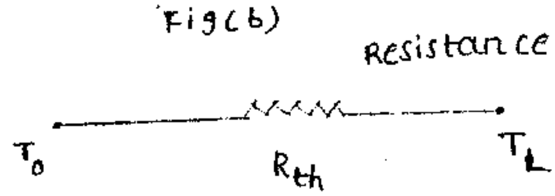


Fig (b)



$$Q = KA \frac{\Delta T}{\Delta x}$$

$$Q = KA \frac{\Delta T}{L} \Rightarrow Q = \frac{KA}{L} \times \Delta T \Rightarrow Q = \frac{\Delta T}{(L/KA)} = \frac{\Delta T}{R_{th}}$$

Thermal conductivity of material

$K_{th} = \frac{KA}{L}$ → Thermal conductance

$$R_{th} = \frac{1}{K} = \frac{1}{\frac{KA}{L}} = \frac{L}{KA}$$ = thermal resi

Convection :-

$$Q = hA\Delta T$$

Convective conductance $K_c = hA$

$$R_c = \frac{1}{hA}$$

$$Q = \frac{\Delta T}{(1/hA)} = \frac{\Delta T}{R_c}$$

Radiation :-

$$Q = \sigma A \epsilon F (T_1^4 - T_2^4)$$

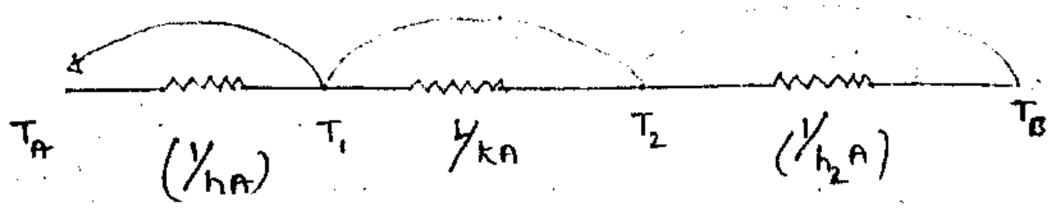
$$Q = \frac{\Delta T}{\left(\frac{1}{\sigma A \epsilon F}\right)} \Rightarrow R_r = \frac{1}{\sigma A \epsilon F}$$

$$Q = [\sigma A \epsilon F (T_1^4 - T_2^4)] \frac{\Delta T}{\Delta T}$$

$$R_r = \frac{\Delta T}{\sigma A \epsilon F (T_1^4 - T_2^4)}$$

$$Q = \frac{\Delta T}{R_r}$$

From Fig (a)



A-1 :-

$$Q = h_1 A (T_A - T_1) \Rightarrow T_A - T_1 = \frac{Q}{h_1 A}$$

1-2 :-

$$Q = KA \frac{[T_1 - T_2]}{L} \Rightarrow (T_1 - T_2) = \frac{Q}{\frac{KA}{L}}$$

2-B :-

$$Q = h_2 A (T_2 - T_B) \Rightarrow (T_2 - T_B) = \frac{Q}{h_2 A}$$

$$\Rightarrow (T_A - T_1 + T_1 - T_2 + T_2 - T_B) = \left[\frac{Q}{h_1 A} + \frac{QL}{KA} + \frac{Q}{h_2 A}\right]$$

$$(T_A - T_B) = Q\left[\frac{1}{h_1 A} + \frac{L}{KA} + \frac{1}{h_2 A}\right]$$

$$Q = \frac{T_A - T_B}{\left[\frac{1}{h_1 A} + \frac{L}{KA} + \frac{1}{h_2 A}\right]} = \frac{\Delta T}{R}$$

$$Q = \frac{1}{\left[\frac{1}{h_1 A} + \frac{L}{KA} + \frac{1}{h_2 A}\right]} \times \Delta T$$

$$Q = \frac{1}{\frac{1}{A}\left[\frac{1}{h_1} + \frac{L}{K} + \frac{1}{h_2}\right]} \times \Delta T$$

$$Q = \frac{1}{\left[\frac{1}{h_1} + \frac{L}{K} + \frac{1}{h_2}\right]} \times A \times \Delta T \Rightarrow Q = UA\Delta T$$

$Q = UA\Delta T$ where $U \rightarrow$ overall heat transfer coefficient

$$U = \frac{1}{\left[\frac{1}{h_1} + \frac{L}{k} + \frac{1}{h_2}\right]}$$

① A constant temperature difference of 166.7°C is maintain across the surfaces of the slab 3.05cm thickness. calculate the rate of heat transfer per unit area across the slab for in each of the following case (i) slab is made up of copper of $k = 380.7\ W/m\text{-}K$ (ii) slab is made of Aluminium of $k = 225\ W/m\text{-}K$ (iii) slab is made of carbon steel of $k = 17.3\ W/m\text{-}K$ (iv) slab is made of Brick of $k = 0.865\ W/m\text{-}K$ (v) slab is made of Asbestos of $k = 0.173\ W/m\text{-}K$

Sol:- given that

thickness of slab $L = 3.05cm = 3.05 \times 10^{-2} m$

temperature difference of slab $\Delta T = 166.7°C = dT$

(i) From Fourier law

$$Q = kA \frac{dT}{dx}$$

$dT \rightarrow$ temperature difference

$dx \rightarrow$ thickness of slab

$K = 380.7\ W/m\text{-}k$

W.K.T $\frac{Q}{A} = \frac{k}{L} \times \Delta T$

$$q = \frac{380.7}{3.05 \times 10^{-2}} \times 166.7 = 2.08 \times 10^{6}\ \frac{W}{m^2}$$

(ii) slab is made of Aluminium of $K = 225\ \frac{W}{m\text{-}k}$

$$\frac{Q}{A} = \frac{k}{L} \Delta T$$

$$q = \frac{225}{3.05 \times 10^{-2}} \times 166.7 = 1.22 \times 10^{6}\ W/m^2 = 1.22\ MW/m^2$$

(iii) slab is made of ~~Aluminium~~ carbon steel of $k = \cancel{205}\ 17.3\ W/m\text{-}k$

$$q = \frac{Q}{A} = \frac{k}{L}\Delta T$$

$$= \frac{17.3}{3.05\times10^{-2}} \times 166.7 = 94.55\ kW/m^2$$

(iv) slab is made of Brick of $k = 0.865\ \frac{W}{m\text{-}k}$

$$q = \frac{Q}{A} = \frac{k}{L}\Delta T$$

$$= \frac{0.865}{3.05\times10^{-2}} \times 166.7$$

$$q = 4.72\ kW/m^2$$

(v) slab is made of Asbestos of $k = 0.173\ W/m\text{-}k$

$$q = \frac{Q}{A} = \frac{k}{L}\Delta T$$

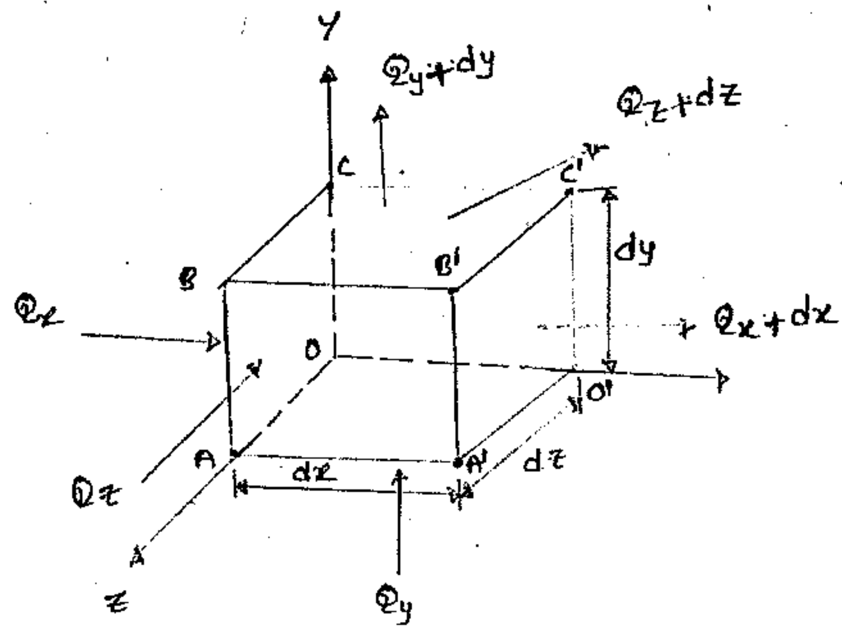
$$= \frac{0.173}{3.05\times10^{-2}} \times 166.7$$

$$q = 945.5\ W/m^2$$

$$q = 0.945\ kW/m^2$$

2) A hot plate of length 0.75m width 0.50m and thickness 2cm is placed in a air stream at 20°c it is estimated that a total of 300w is lost from plate surface by the convective heat transfer coefficient as radiation taken

General heat conduction equation in cartesion coordinate system



(heat conducted in body dx dy dz per unit time) + (heat generated with in the body per unit time)

= (change in internal energy per unit time) + [work done by the system per unit time]

At ABCO:- x-direction

$$Q_x = -k_x(dy\,dz) \times \frac{\partial T}{\partial x}\,dt$$

$$Q_{x+dx} = \left[-k_x\,(dy\,dz)\,\frac{\partial T}{\partial x}\,dt\right] + \frac{\partial}{\partial x}\left[-k_x(dy\,dz)\,\frac{\partial T}{\partial x}\,dt\right]dx$$

y-direction :

$$Q_y = -k_y\,[dx \times dz]\,\frac{\partial T}{\partial y}\,dt$$

$$Q_{y+dy} = Q_y + \frac{\partial}{\partial y}(Q_y)\,dy = -k_y[dx\,dz]\frac{\partial t}{\partial y} \times dt + \frac{\partial}{\partial y}\left[-k_y\,(dx\,dz)\,\frac{\partial t}{\partial y}\,dt \times dy\right]$$

z-direction :-

$$Q_z = -k_z \, dx \, dy \times \frac{\partial T}{\partial z} \times dt$$

$$Q_{z+dz} = \left[-k_z \, dx \, dy \frac{\partial T}{\partial z} dt\right] + \frac{\partial}{\partial z}\left[-k \, dz \, dy \frac{\partial T}{\partial z} dt \, dz\right]$$

Heat conducted $= \left[Q_x - Q_{x+dx}\right] + \left[Q_y - Q_{y+dy}\right] +$

$$\left[Q_z - Q_{z+dz}\right]$$

$$= \left\{\left[-k_x \, dy \, dz \frac{\partial T}{\partial x} dt\right] - \left\{\left[\left(-k_x \, dy \, dz \frac{\partial T}{\partial x} dt\right]\right\}\right.$$

$$+ \left[-\frac{\partial}{\partial x} k_x (dy \, dz) \frac{\partial T}{\partial x} dt\right] dx \Big\} \Rightarrow k_x' \frac{\partial^2 T}{\partial x^2} x$$

$$dx \, dy \, dz \, dt + k_y \frac{\partial^2 T}{\partial y^2} dx \, dy \, dz \, dt + k_z \frac{\partial^2 T}{\partial z^2} dx \, dy \, dz \, dt$$

$$\dot{q} = \frac{Q}{dt}$$

$\dot{q} \times dx \, dy \, dz \, dt \longrightarrow$ heat generated with in the body per unit time

* change in internal energy $= m c_p \Delta T = m c_p \, dT$

$$= \rho \times V \times c_p \times dT$$

$$= \rho \times dx \, dy \, dz \times c_p \times \frac{\partial T}{\partial t}$$

$$= \left[k_x \frac{\partial T^2}{\partial x^2} \frac{(dx \, dy \, dz) \, dt}{dt} + k_y \frac{\partial T^2}{\partial y^2} \frac{(dx \, dy \, dz) \, dt}{dt} \right.$$

$$\left. + k_z \frac{\partial T^2}{\partial z^2} \frac{(dx \, dy \, dz) \, dt}{dt} \right] + \frac{\dot{q}(dx \, dy \, dz) \, dt}{dt}$$

$$= \frac{\rho c_p \, \partial T \, dx \, dy \, dz}{\partial t}$$

$$\left[k_x \frac{\partial^2 T}{\partial x^2} + k_y \frac{\partial^2 T}{\partial y^2} + k_z \frac{\partial^2 T}{\partial z^2}\right] + \dot{q} = \rho C_p \frac{\partial T}{\partial t}$$

General heat conduction equation for non-homogenous, anisotropic unsteady, with internal heat generated

if $k_x = k_y = k_z = k$ (Transiant condition and heat generated eqn)

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}}{k} = \frac{\rho C_p}{k} \frac{\partial T}{\partial t} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

where α = thermal diffusivity $= \frac{k}{\rho C_p}$

(i) steady state conduction :- $\frac{\partial T}{\partial t} = 0$

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}}{k} = 0 \longrightarrow \text{poison equation}$$

(ii) without internal heat generated $\frac{\partial T}{\partial t} \neq 0$

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \longrightarrow \text{diffusivity equation}$$

(iii) no heat generation, steady state

$$\left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right] = 0 \longrightarrow \text{Laplace equation}$$

$$\nabla^2 T = 0$$

(iv) For 1-D, steady, no heat generation

$$\frac{\partial^2 T}{\partial x^2} = 0$$

General heat Conduction equation in cylindrical co-ordinate system :-

$T = (r, \phi, z, t)$

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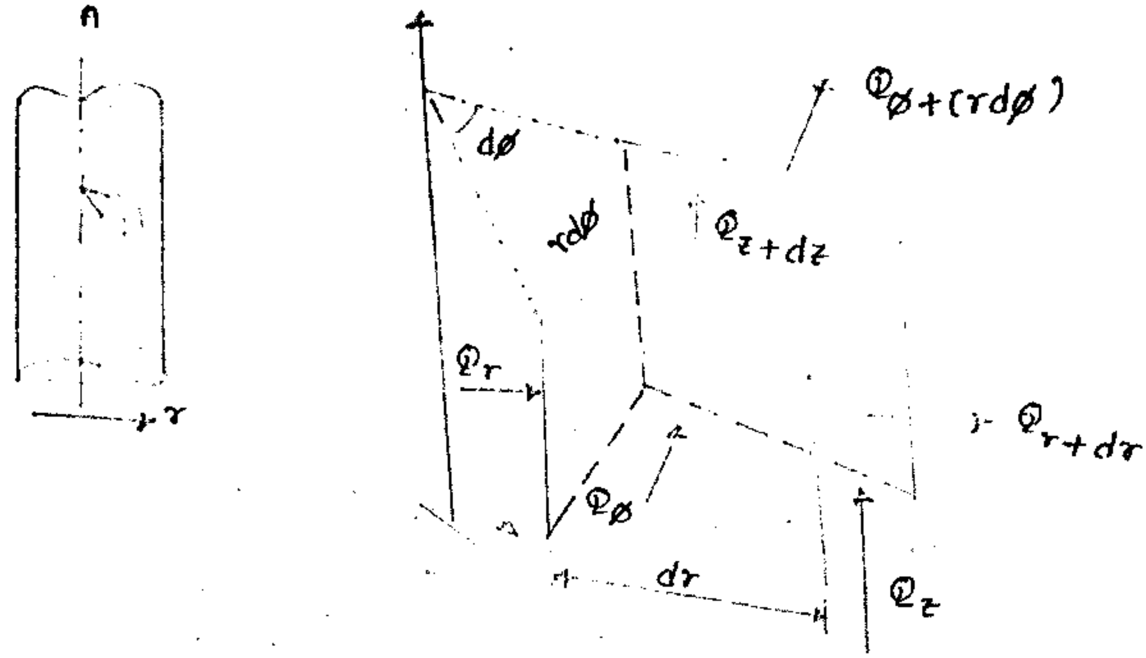

[heat conduct in body of volume per unit time + heat generated with in the body per unit time]

= [change in internal body per unit time + work done per unit time]

<u>In r-direction</u> <u>at "r"</u>

$$Q_r = -k_r(rd\phi\,dz) \times \frac{\partial T}{\partial r}$$

<u>at r+dr</u>

$$Q_{r+dr} = Q_r + \frac{\partial[Q_r]}{\partial r} \times dr$$

$$= -k_r(rd\phi dz)\frac{\partial T}{\partial r} + \frac{\partial}{\partial r}\left[-k_r\, r\,d\phi\, dz\ \frac{\partial T}{\partial r}\right] dr$$

<u>Net heat conducted in r - direction</u> $= Q_r - (Q_r + \frac{\partial}{\partial r}(Q_r)dr$

$$[Q_r - Q_{r+dr}] = \left[-k_r(rd\phi dz) \times \frac{\partial T}{\partial r} - \left(-k_r(rd\phi dz)\frac{\partial T}{\partial r} + \frac{\partial}{\partial r}\left[-k_r\, rd\phi\, dz\left(\frac{\partial T}{\partial r}\right] dr\right)\right]$$

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ϕ-direction at ϕ

$$Q_\phi = -k_\phi \,(dr\,dz) \times \frac{\partial T}{r\partial \phi}$$

at $(r d\phi)$

$$Q_{\phi + r d\phi} = Q_\phi + \frac{\partial}{r\partial\phi}(Q_\phi) \times r\,d\phi \quad = -k_\phi\,(dr\,dz)\frac{\partial T}{r\partial\phi}$$

$$+ \frac{\partial}{r\partial\phi}\left[-k_\phi (dr\,dz)\frac{\partial T}{r\partial\phi}\right] r\,d\phi$$

net heat conducted in ϕ

$$Q_\phi - Q_{\phi + r d\phi} = -k_\phi\,(dr\,dz) \times \frac{\partial T}{r\partial\phi} - \left[-k_\phi (dr\,dz)\frac{\partial T}{r\partial\phi} + \right.$$

$$\left.\frac{\partial}{r\partial\phi}\left[-k_\phi (dr\,dz)\frac{\partial T}{r\partial\phi}\right] r\,d\phi\right]$$

$$= \frac{\partial}{r\partial\phi}\left[k_\phi\,(dr\,dz)\frac{\partial T}{r\partial\phi}\right] \times r\,d\phi$$

$$= k_\phi\,[(r\,dr\,dz\,d\phi)] \times \frac{\partial}{r\partial\phi}\left(\frac{1}{r}\frac{\partial T}{\partial\phi}\right)$$

$$= k_\phi\,(r\,dr\,d\phi\,dz)\cdot\frac{1}{r^2}\frac{\partial^2 T}{\partial\phi^2}$$

Net heat conducted in r

$$Q_r - Q_{r+dr} = \frac{\partial}{\partial r}\left[k_r\; r\,(d\phi\,dz)\frac{\partial T}{\partial r}\right] dr \;= \frac{\partial}{\partial r}\left[r\,\frac{\partial T}{\partial r}\right] dr\,d\phi\,dz \times k_r$$

$$= \left[\frac{\partial}{\partial r}(r)\,\frac{\partial T}{\partial r} + \frac{\partial}{\partial r}\left(\frac{\partial T}{\partial r}\right) r\right] k_r\,dr\,d\phi\,dz$$

$$= \left(\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial r^2} \times r\right] k_r\,dr\,d\phi\,dz$$

$$Q_r - Q_{r+dr} = k_r\left[\frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial r^2}\right] (r\,dr\,d\phi\,dz)$$

z-direction :-

$$Q_z = -k_z\,(dr \times d\phi\,r)\,\frac{\partial T}{\partial z}$$

at z+dz :-

$$Q_{z+dz} = -k_z\,[dr \times r\,d\phi)\,\frac{\partial T}{\partial z} + \frac{\partial}{\partial z}\left[-k_z\,(dr \times r\,d\phi)\frac{\partial T}{\partial z}\right] dz$$

net heat conducted in z

$$Q_z - Q_{z+dz} = k_z\,(r\,dr\,d\phi\,dz)\,\frac{\partial^2 T}{\partial z^2}$$

Add all equations

$$k_r\left[\frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial r^2}\right](r\,dr\,d\phi\,dz) + k_\phi\,(r\,dr\,d\phi\,dz)$$

$$\frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2} + k_z\,(r\,dr\,d\phi\,dz)\,\frac{\partial^2 T}{\partial z^2} \longrightarrow (1)$$

heat generated $= \dot{q}\,(r\,dr\,d\phi\,dz) \longrightarrow (2)$

change in internal energy $= \rho \times (r\,dr\,d\phi\,dz) \times C_p \times \frac{\partial T}{\partial t} \longrightarrow (3)$

General heat conduction equation $k_r = k_\phi = k_z = k$

$$k\left(\frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2} + \frac{\partial^2 T}{\partial z^2}\right] r\,dr\,d\phi\,dz +$$

$$\dot{q}\,(r\,dr\,d\phi\,dz) = \rho\,C_p\,\frac{\partial T}{\partial t}\,r \times dr\,d\phi\,dz$$

$$\Rightarrow \left[\frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2} + \frac{\partial^2 T}{\partial z^2}\right] + \frac{\dot{q}}{k} = \frac{\rho C_p}{k} \times \frac{\partial T}{\partial t}$$

$$= \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

$$\boxed{\alpha = \frac{k}{\rho C_p}}$$

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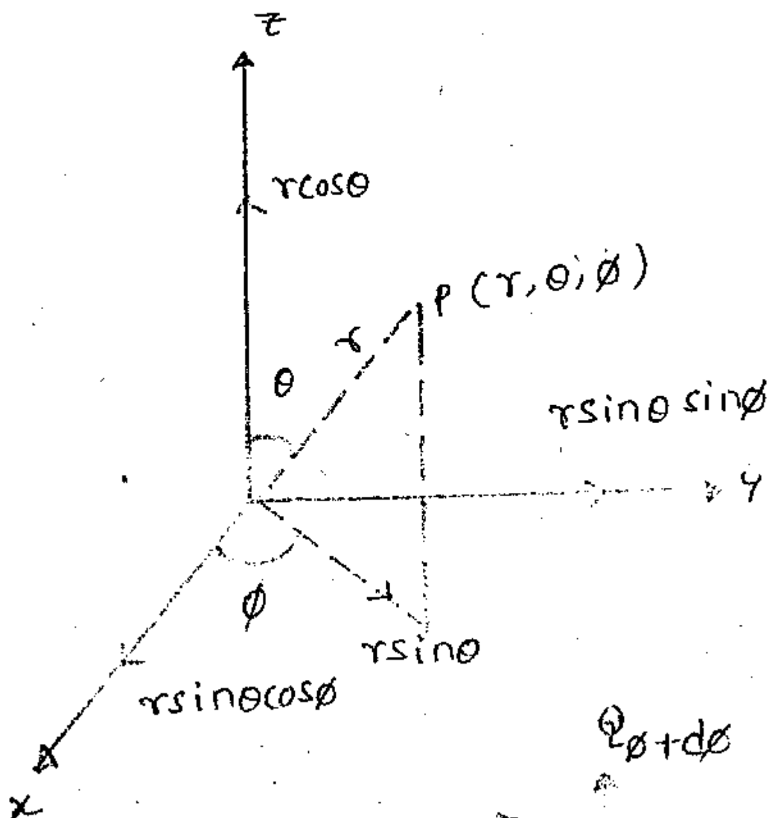
Heat conduction equation in spherical coordinates

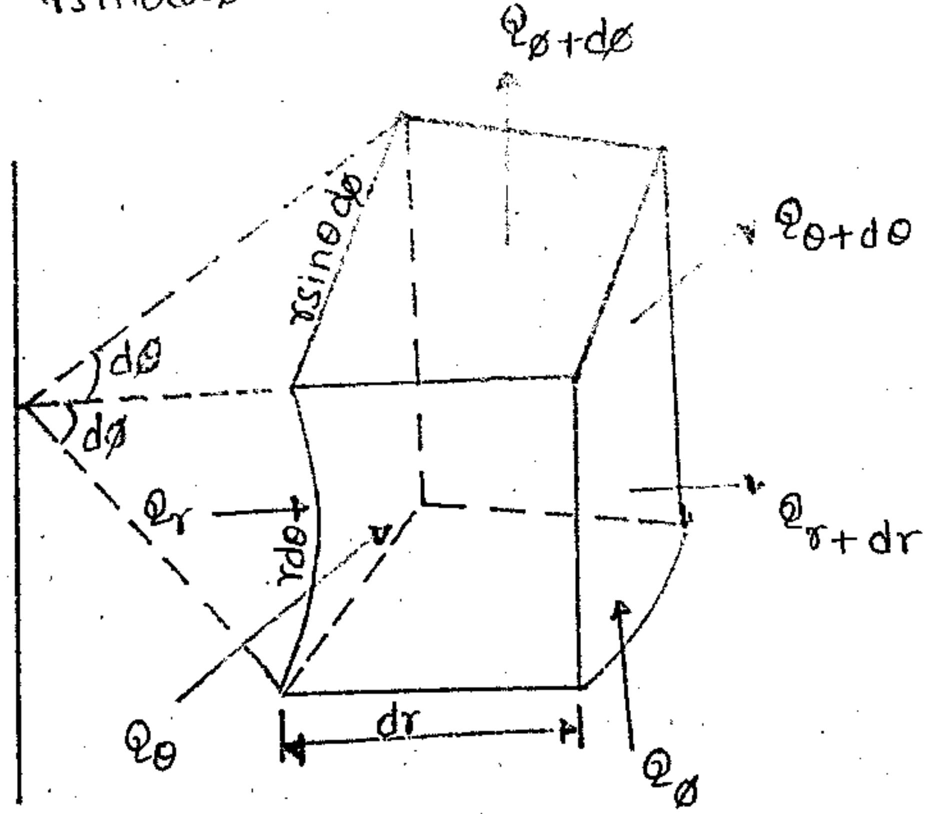
Consider a small element of sphere having the coordinates (r, θ, ϕ). Let the dimensions of sphere be dr, $r\sin\theta\, d\phi$ and $r\, d\theta$.



$x = r\sin\theta\cos\phi$

$y = r\sin\theta\sin\phi$

$z = r\cos\theta$



In r-direction at r

let Q_r be the heat entering into element in radial direction

$$Q_r = -k_r[rd\theta \times r\sin\theta d\phi] \times \frac{\partial T}{\partial r}$$

at $r+dr$

Q_{r+dr} be the heat leaving the element at $(r+dr)$ in radial direction

$$Q_{r+dr} = -k_r[rd\theta \times r\sin\theta d\phi] \times \frac{\partial T}{\partial r} + \frac{\partial}{\partial r}\left[-k_r(rd\theta \times r\sin\theta d\phi) \times \frac{\partial T}{\partial r}\right] dr$$

Net heat conducted in radial "r" direction

$$dQ_r = Q_r - Q_{r+dr}$$

$$= -k_r[rd\theta \times r\sin\theta d\phi] \times \frac{\partial T}{\partial r} - \left\{-k_r(rd\theta \times r\sin\theta d\phi) \times \frac{\partial T}{\partial r} + \frac{\partial}{\partial r}\left[-k_r(rd\theta \times r\sin\theta d\phi) \times \frac{\partial T}{\partial r}\right] dr\right\}$$

$$dQ_r = Q_r - Q_{r+dr} = k_r \frac{\partial}{\partial r}\left[r^2 \frac{\partial T}{\partial r}\right] dr \sin\theta \times d\theta \times d\phi$$

$$= k_r\left[r^2 \frac{\partial^2 T}{\partial r^2} + 2r\frac{\partial T}{\partial r}\right] dr\sin\theta \times d\theta \times d\phi$$

$$= k_r\left[\frac{\partial^2 T}{\partial r^2} + \frac{2}{r}\frac{\partial T}{\partial r}\right] \times r^2 \times dr\sin\theta\, d\theta \times d\phi$$

$$= k_r\left[\frac{\partial^2 T}{\partial r^2} + \frac{2}{r}\frac{\partial T}{\partial r}\right] \times (r^2 dr\sin\theta\, d\theta \times d\phi)$$

In θ-direction at θ

Let Q_θ be heat conducted per unit time in "θ" direction

$$Q_\theta = -k_\theta \times (dr \times r d\theta) \times \frac{\partial T}{\partial (r \sin\theta \, d\phi)}$$

at $\theta + d\theta$:-

$Q_{\theta+d\theta}$ be the heat leaving the element in θ direction is

$$Q_{\theta+d\theta} = -k_\theta [dr \times r d\theta] \times \frac{\partial T}{\partial (r\sin\theta \, d\phi)} + \frac{\partial}{\partial (r\sin\theta \, d\phi)} \left[-k_\theta (dr \times r d\theta) \times \frac{\partial T}{\partial (r\sin\theta \, d\phi)} \right] \times r\sin\theta \, d\phi$$

Net heat conducted into element in "θ" direction is

$$dQ_\theta = Q_\theta - Q_{\theta+d\theta}$$

$$= -k_\theta [dr \times r d\theta] \times \frac{\partial T}{\partial (r\sin\theta \, d\phi)} - \Big\{ -k_\theta (dr \times r d\theta) \times \frac{\partial T}{\partial (r\sin\theta \, d\phi)} + \frac{\partial}{\partial (r\sin\theta \, d\phi)} \left[-k_\theta (dr \times r d\theta) \times \frac{\partial T}{\partial (r\sin\theta \, d\phi)} \right] \times r\sin\theta \, d\phi \Big\}$$

$$dQ_\theta = Q_\theta - Q_{\theta+d\theta} = k_\theta \times (dr \times r d\theta) \times r\sin\theta \, d\phi \times \frac{\partial}{\partial (r\sin\theta \, d\phi)} \times \left[\frac{\partial T}{\partial (r\sin\theta \, d\phi)} \right]$$

$$dQ_\theta = Q_\theta - Q_{\theta+d\theta} = k_\theta \times \frac{\partial^2 T}{[\partial (r\sin\phi \, d\phi)]^2} \times r^2 \sin\theta \, d\theta \times dr \times d\phi$$

In ϕ - direction at ϕ

Let Q_ϕ be the heat conducted per unit time in ϕ direction

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$$Q_\phi = -k_\phi\,[dr \times r\sin\theta \times d\phi]\,\frac{\partial T}{\partial(rd\theta)}$$

at $\phi + d\phi$

$Q_{\phi+d\phi}$ be the heat leaving the element in ϕ direction.

$$Q_{\phi+d\phi} = -k_\phi\,[dr \times r\sin\theta \times d\phi]\,\frac{\partial T}{\partial(rd\theta)} + \frac{\partial}{\partial(rd\theta)}\left[-k_\phi\,(dr \times r\sin\theta \times d\phi)\,\frac{\partial T}{\partial(rd\theta)}\right] \times rd\theta$$

Net heat conducted into element in ϕ direction

$$dQ_\phi = Q_\phi - Q_{\phi+d\phi}$$

$$= -k_\phi\,[dr \times r\sin\theta \times d\phi]\,\frac{\partial T}{\partial(rd\theta)} - \left\{-k_r\,(dr \times r\sin\theta \times d\phi) \times \frac{\partial T}{\partial(rd\theta)} + \frac{\partial T}{\partial(rd\theta)}\left[-k_\phi\,(dr \times r\sin\theta \times d\phi)\,\frac{\partial T}{\partial(rd\theta)}\right] \times rd\theta\right\}$$

$$dQ_\phi = Q_\phi - Q_{\phi+d\phi} = k_\phi\,\frac{\partial^2 T}{[\partial(rd\theta)]^2} \times [dr\,d\theta \times r^2\sin\theta \times d\phi]$$

let $q^\cdot$ be the internal energy generated per unit volume in the element of element so the internal energy generated is

heat generated $= q^\cdot \times r^2 dr\sin\theta\,d\theta \times d\phi$

change in internal energy $= mC_p\,\frac{\partial T}{\partial t} = \rho \times V\,C_p \times \frac{\partial T}{\partial t}$

$$= \rho \times (r^2 dr\sin\theta\,d\theta \times d\phi) \times \frac{\partial T}{\partial t}$$

General heat conducted equation is

$k_r = k_\theta = k_\phi = k$ $\longrightarrow$ This is Transient condition

for spherical coordinates

$$\left[\frac{\partial^2 T}{\partial r^2} + \frac{2}{r}\frac{\partial T}{\partial r}\right] + \frac{\partial^2 T}{[\partial(r\sin\theta\, d\phi)]^2} + \frac{\partial^2 T}{[\partial(r\, d\theta)]^2} + \frac{q^\cdot}{k}$$

$$= \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

where $\frac{1}{\alpha} \rightarrow \frac{\rho C_p}{k} \rightarrow$ thermal diffusivity

For steady state one dimensional heat conduction in radial direction without heat generation, the above equation takes the form

here $q^\cdot = 0$ no heat sources present

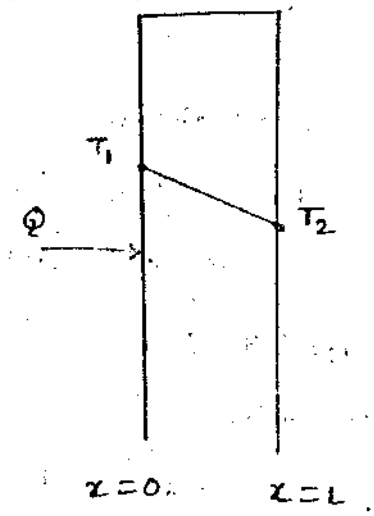
$$\frac{\partial^2 T}{\partial r^2} + \frac{2}{r}\frac{\partial T}{\partial r} = 0$$

$$\frac{\partial}{\partial r}\left[r^2 \frac{\partial T}{\partial r}\right] = 0$$

one dimensional steady state heat conduction for plane wall

$$Q = -kA\frac{dT}{dx}$$

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$



$\frac{\partial T}{\partial t} = 0$ (steady state)

$$\frac{\partial^2 T}{\partial y^2} = \frac{\partial^2 T}{\partial z^2} = 0$$

Consider 1-D system

$$\int \frac{\partial^2 T}{\partial x^2} = \int 0 \longrightarrow (1)$$

Integrate eq (1) w.r.to "x"

$$\int \frac{\partial T}{\partial x} = \int c_1 \longrightarrow (2)$$

Integrate eq (2)

$$T = c_1 x + c_2 \longrightarrow (3)$$

Boundary conditions

at $x = 0$, $T = T_1$

$x = L$, $T = T_2$

at $x = 0 \Rightarrow T_1 = 0 + c_2 \Rightarrow T_1 = c_2$

$x = L \Rightarrow T_2 = c_1 L + c_2 \Rightarrow T_2 = c_1 L + T_1$

$$\frac{(T_2 - T_1)}{L} = c_1$$

from eq (3) $T = \frac{(T_2 - T_1)}{L}x + T_1$ (Temperature at "x" distance)

$$\frac{dT}{dx} = \frac{(T_2 - T_1)}{L}$$

$$Q = -kA\frac{(T_2 - T_1)}{L}$$

$$Q = kA\frac{(T_1 - T_2)}{L}$$

1) calculate the rate of heat loss for a red brick wall of length 5m, height 4m and thickness 0.25m. the temperature of the inner surface is 110°c and that of outer surface is 40°c. the thermal conductivity of the red brick, k = 0.71 W/m-k. calculate also the temperature of the at the interior point of the wall 20cm distance from the innerwall

1:- given that

① $Q = kA\frac{dT}{dx}$

$$= 0.71\ W/mK \times (4 \times 5)\ \frac{[(110+273)-(40+273)]}{0.25}$$

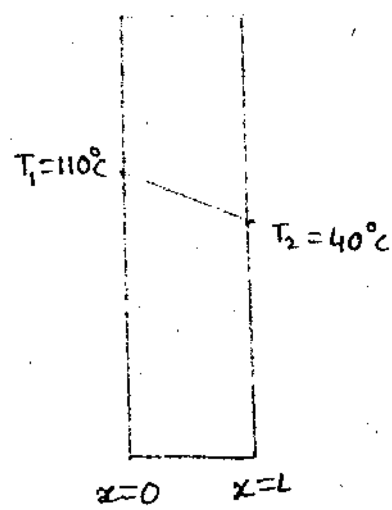
$$= 3.97\,kW$$

② at $x = 0.2m$

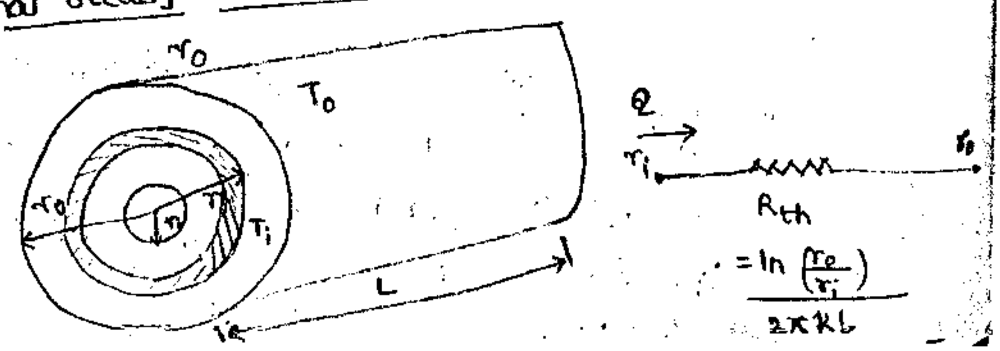
$$T = \frac{(T_2 - T_1)}{L}\,x$$

$$T = \frac{(40-110)}{0.25} \times 0.2$$

$$T = 54°c$$



one dimensional steady state heat conductivity (Radial)



$$\frac{1}{r}\left[\frac{\partial T}{\partial r}\right] + \frac{\partial^2 T}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2} + \frac{\partial^2 T}{\partial z^2} + \frac{q^\circ}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

↳ General heat conduction eqn in cylindrical coordinate system

$$\frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2} = 0 = \frac{\partial^2 T}{\partial z^2} \qquad \frac{q^\circ}{k} = 0, \quad \frac{\partial T}{\partial t} = 0$$

$$\frac{1}{r}\left[\frac{\partial T}{\partial r}\right] + \frac{\partial^2 T}{\partial r^2} = 0 \Rightarrow \frac{1}{r}\left[\frac{dT}{dr}\right] + \frac{d^2 T}{dr^2} = 0$$

$$\frac{1}{r}\left[\frac{dT}{dr}\right] + \frac{d}{dr}\left[\frac{dT}{dr}\right] = 0$$

$$\frac{d}{dr}\left[r\frac{dT}{dr}\right] = 0$$

Integrating $r\dfrac{dT}{dr} = C_1$

$$\frac{dT}{dr} = \frac{C_1}{r} \longrightarrow (2)$$

Integrating (2) $\Rightarrow T = C_1 \ln r + C_2$

$$T = C_1 \ln r + C_2$$

Boundary conditions

at $r = r_i \Rightarrow T = T_i$

$r = r_o \Rightarrow T = T_o$

at $r = r_i \Rightarrow T_i = C_1 \ln r_i + C_2 \Rightarrow C_2 = T_i - C_1 \ln r_i$

$r = r_o \Rightarrow T_o = C_1 r_o \ln + C_2$

$$T_o = C_1 \ln r_o + T_i - C_1 \ln r_i$$

$$(T_o - T_i) = C_1(\ln r_o - \ln r_i) \Rightarrow C_1\left(\ln\left(\frac{r_o}{r_i}\right)\right) = T_o - T_i$$

$$C_1 = \frac{T_o - T_i}{\ln\left(\frac{r_o}{r_i}\right)}$$

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$$T = \frac{(T_0 - T_i)}{\ln\left(\frac{r_0}{r_i}\right)} \ln r + T_i - \frac{(T_0 - T_i)}{\ln\left(\frac{r_0}{r_i}\right)} \ln r_i$$

$$T = \frac{(T_0 - T_i)}{\ln\left(\frac{r_0}{r_i}\right)} \ln r + \frac{T_i}{1} - \frac{(T_0 - T_i)}{\ln\left(\frac{r_0}{r_i}\right)} \ln r_i$$

$$= \frac{(T_0 - T_i)\ln r + T_i \ln\left(\frac{r_0}{r_i}\right) - (T_0 - T_i)\ln r_i}{\ln\left(\frac{r_0}{r_i}\right)}$$

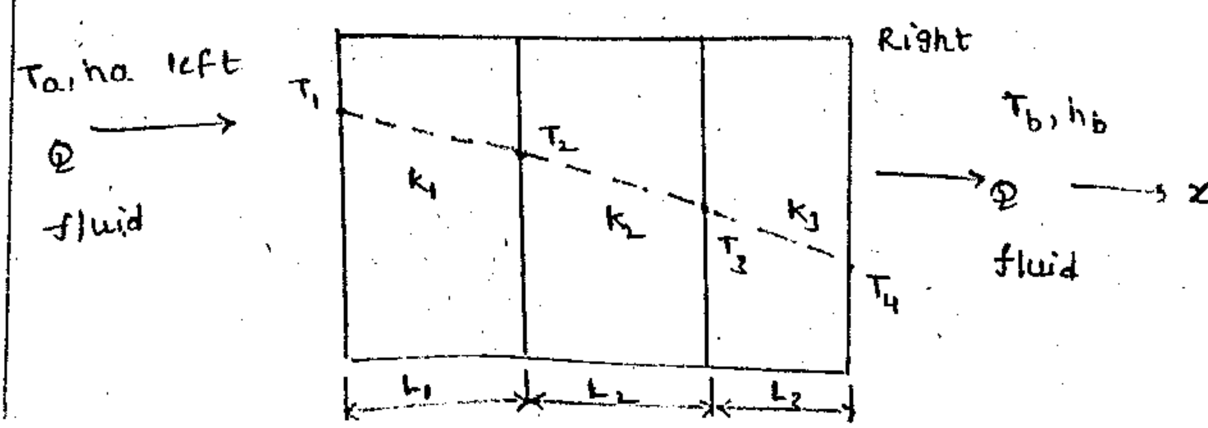
$$T = \frac{T_0 \ln r - T_i \ln r + T_i \ln r_0 - T_i \ln r_i - T_0 \ln r_i + T_i \ln r_i}{\ln\left(\frac{r_0}{r_i}\right)}$$

$$T = \frac{T_0[\ln r - \ln r_i] - T_i[\ln r - \ln r_0]}{\ln\left(\frac{r_0}{r_i}\right)}$$

$$T = \frac{T_0 \ln\left[\frac{r}{r_i}\right] - T_i\left[\ln \times \frac{r}{r_0}\right]}{\ln\left[\frac{r_0}{r_i}\right]}$$

Heat transfer through composite slab

Two (or) more different material combined together and treated as a single body



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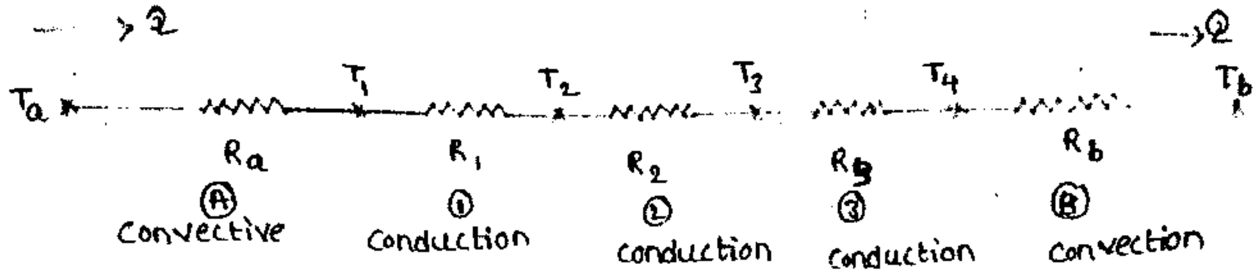


Assumptions :-

(1) Rate of heat transfer is takes place only in steady state

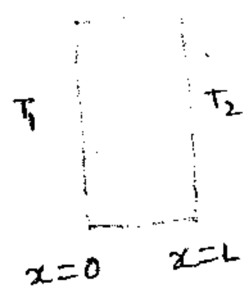
(2) material is homogeneous

(3) No internal heat generation

(4) heat flow is one-dimensional

$$\frac{\partial^2 T}{\partial x^2} + \underbrace{\frac{\partial^2 T}{\partial y^2}}_{=0} + \underbrace{\frac{\partial^2 T}{\partial z^2}}_{=0} + \underbrace{\frac{q^{\circ}}{K}}_{=0} = \frac{1}{\alpha} \underbrace{\frac{\partial T}{\partial t}}_{=0}$$

$$\frac{\partial^2 T}{\partial x^2} = 0 \Rightarrow \frac{\partial T}{\partial x} = C_1 \Rightarrow T = C_1 x + C_2$$



$x=0, \ T=T_1$

$x=L, \ T=T_2$

$$T = \frac{(T_1 - T_2)}{L} x + T_1$$

$$Q = \frac{\Delta T_{overall}}{R}$$

$$R = R_a + R_1 + R_2 + R_3 + R_b$$

at A $\Rightarrow Q = h_a A (T_a - T_1) \Rightarrow Q = \frac{(T_a - T_1)}{1/h_a A} = \frac{\Delta T}{R_{th}}$

at B $\Rightarrow Q = h_b A (T_4 - T_b) \Rightarrow Q = \frac{(T_4 - T_b)}{1/h_b A} = \frac{\Delta T}{R_{th}}$

<u>Newton's law of cooling</u> $\rightarrow$ mode of convection heat transfer

heat transfer rate is directly proportional to the surface area of heat flow of the temperature difference (flow of fluid and surface)

$$Q \propto A(T_s - T_a) \Rightarrow Q = h(T_s - T_a) \times A$$

in slab ①

$$Q = -kA\frac{dT}{dx} = -kA \times \frac{(T_2 - T_1)}{L_1} \Rightarrow Q = k_1 A\frac{(T_1 - T_2)}{L_1}$$

$$Q = \frac{(T_1 - T_2)}{(L_1/k_1 A)} = \frac{\Delta T}{R_{th}} \Rightarrow R_1$$

$$R_a = \frac{1}{h_a A}$$

$$R_b = \frac{1}{h_b A}$$

slab ②

$$Q = \frac{(T_2 - T_3)}{(L_2/k_2 A)} = R_2$$

$$R_1 = \frac{L_1}{k_1 A}$$

$$R_2 = \frac{L_2}{k_2 A}$$

slab ③

$$Q = \frac{(T_3 - T_4)}{(L_3/k_3 A)}$$

$$R_3 = \frac{L_3}{k_3 A}$$

$$R = \frac{1}{h_a A} + \frac{L_1}{k_1 A} + \frac{L_2}{k_2 A} + \frac{L_3}{k_3 A} + \frac{1}{h_b A}$$

equivalent thermal resistance

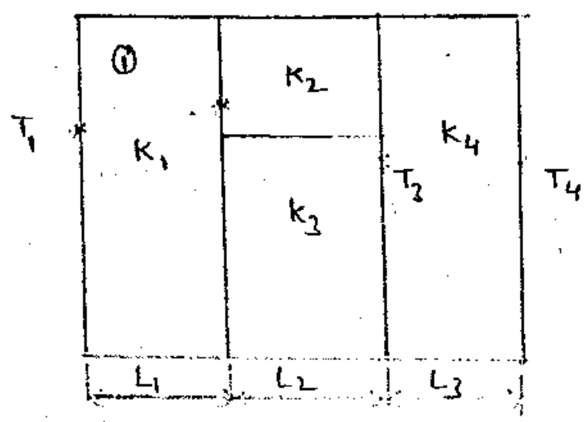
$$R = \frac{1}{A}\left[\frac{1}{h_a} + \frac{L_1}{k_1} + \frac{L_2}{k_2} + \frac{L_3}{k_3} + \frac{1}{h_b}\right]$$

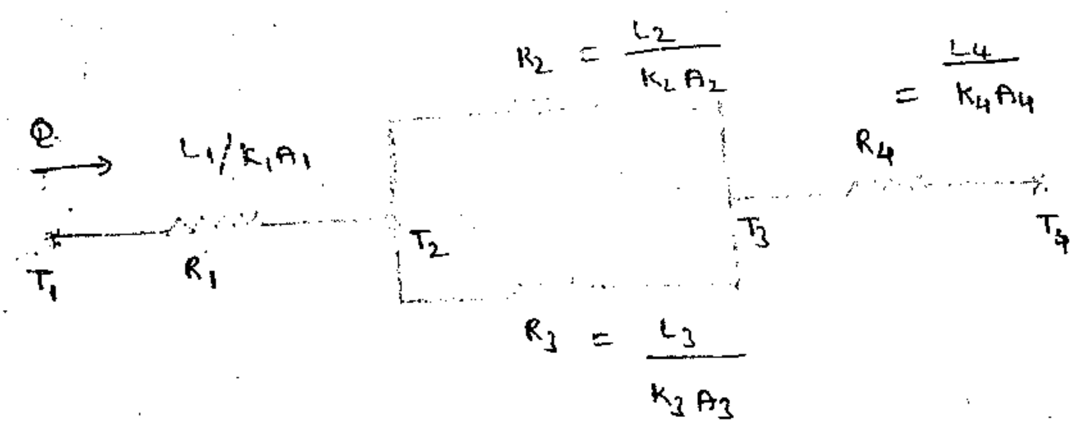
$$Q = \frac{\Delta T_{overall}}{R} = \frac{(T_a - T_1) + (T_1 - T_2) + (T_2 - T_3) + (T_3 - T_4) + (T_4 - T_b)}{R_{eq}}$$

$$Q = \frac{(T_a - T_b)}{\left[\frac{1}{A}\left(\frac{1}{h_a} + \frac{L_1}{k_1} + \frac{L_2}{k_2} + \frac{L_3}{k_3} + \frac{1}{h_b}\right)\right]}$$

$$Q = \frac{1}{\left[\frac{1}{h_a} + \frac{L_1}{k_1} + \frac{L_2}{k_2} + \frac{L_3}{k_3} + \frac{1}{h_b}\right]} \times A\,(T_a - T_b) = UA\Delta T$$

U → overall heat transfer coefficient $= \dfrac{1}{\left(\frac{1}{h_a} + \frac{L_1}{k_1} + \frac{L_2}{k_2} + \frac{L_3}{k_3} + \frac{1}{h_b}\right)}$





$$R = R_1 + \left[\frac{1}{R_2} + \frac{1}{R_3}\right] + R_4$$

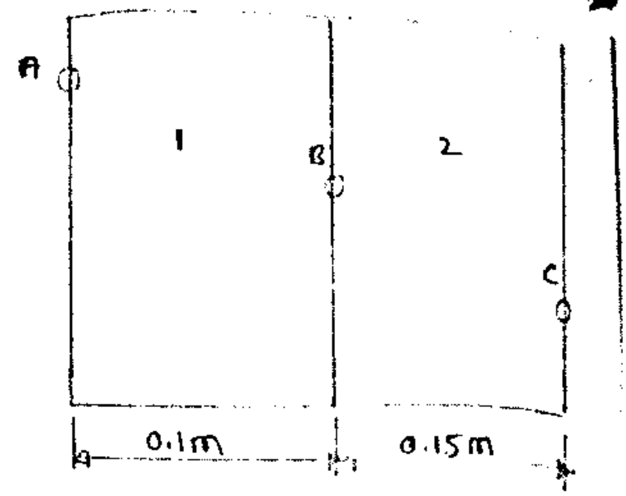
<u>composite slab problem</u>

① heat is flowing through composite wall at rate of 7800 W/m² as shown in the fig. If the left face temperature is 168°c and right face temperature is 38°c. conductivity ratio of slab 2 to slab 1 is 0.75 Determine

① Temperature at interface B

② Ratio of thermal resistances of slab 1 to slab 2

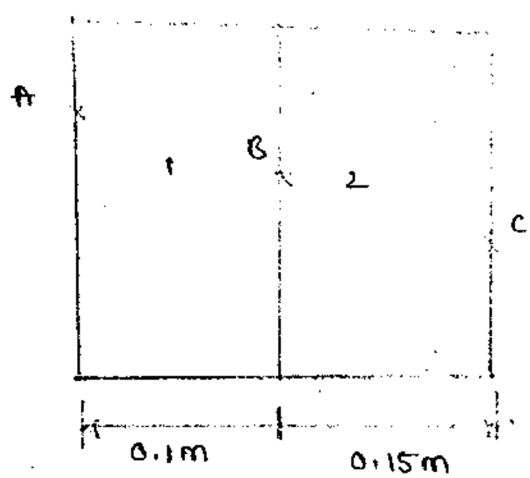
③ Temperature gradient ratio of slab 2 to slab 1



Sol:- given that

Composite wall rate $q = 7800\, W/m^2$, Temperature $T_A = 168°c$, $T_B = ?$

$T_c = 68°c$, Conductivity ratio $\frac{k_2}{k_1} = 0.75$



$Q \rightarrow$ in slab 1 $= Q$ in slab 2

$$\frac{\Delta T_1}{R_1} = \frac{\Delta T_2}{R_2}$$

$$\frac{(T_A - T_B)}{R_1} = \frac{(T_B - T_C)}{R_2}$$

① Temperature at interface B

$$Q = k_1 A_1 \frac{\Delta T_1}{L_1} = k_1 A_1 \frac{(T_A - T_B)}{L_1}$$

$$\frac{Q}{A} = \frac{k_1}{L_1}(T_A - T_B)$$

$\boxed{A_1 = A_2}$

$$\frac{(T_A - T_B)}{\frac{L_1}{k_1 A_1}} = \frac{(T_B - T_c)}{\frac{L_2}{k_2 A_2}}$$

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$$\frac{k_1}{L_1}(T_A - T_B) = (T_B - T_C) \times \frac{k_2}{L_2} \qquad \boxed{k_2 = 0.75\,k_1}$$

$$\frac{\cancel{k_1}}{0.1}[168 - T_B] = [T_B - 38] \times \frac{0.75\cancel{k_1}}{0.15}$$

$$(168 - T_B) = (T_B - 38) \times \frac{0.75}{0.15} \times 0.1$$

$$(168 - T_B) = (T_B - 38) \times 0.5$$

$$168 - T_B = 0.5T_B - 38 \times 0.5 \Rightarrow 1.5T_B = 168 + 0.5 \times 38$$

$$\Rightarrow T_B = 124.66°C$$

$$T_B = 124.66°C + 273 = 397.66\,K$$

(2) Ratio's of thermal resistance

$$\frac{R_1}{R_2} = \frac{\frac{L_1}{k_1 A}}{\frac{L_2}{k_2 A}} = \frac{\frac{0.1}{k_1}}{\frac{0.15}{0.75\,k_1}} = \frac{0.1 \times 0.75}{0.15} = 0.5$$

$$\frac{R_1}{R_2} = 0.5$$

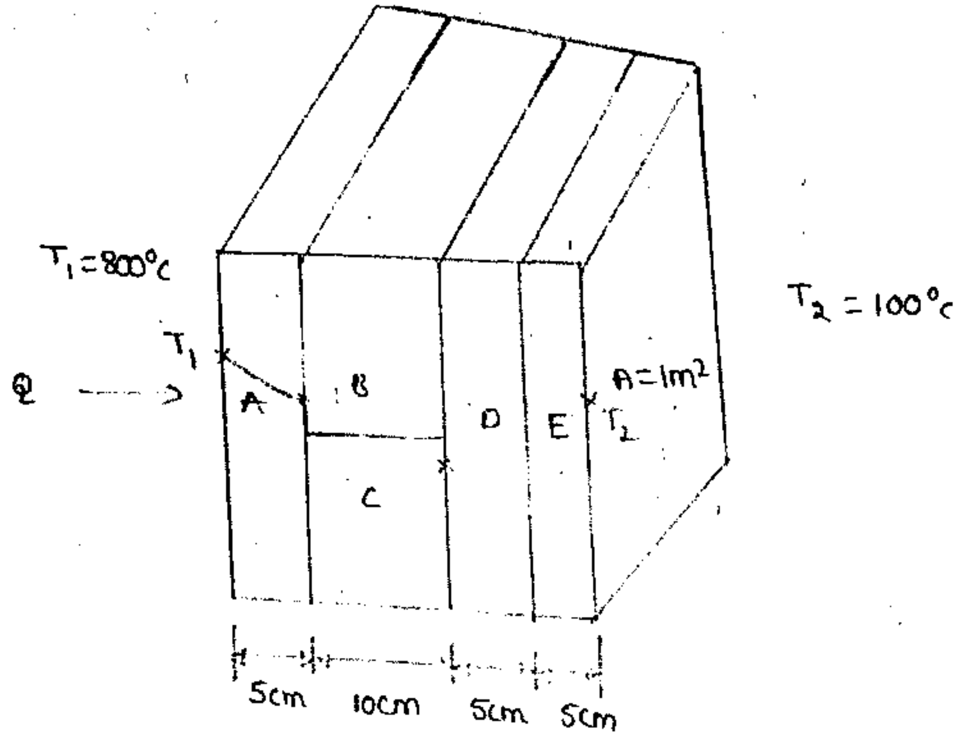
(3) temperature gradient ratio of slab 2 to slab 1

$$\frac{\Delta T_1}{R_1} = \frac{\Delta T_2}{R_2} \Rightarrow \frac{\Delta T_2}{\Delta T_1} = \frac{R_2}{R_1} = \frac{1}{0.5} = 2$$

$$\frac{\Delta T_2}{\Delta T_1} = 2 \Rightarrow \Delta T_2 = 2\Delta T_1$$

$$\Delta T_2 > \Delta T_1$$

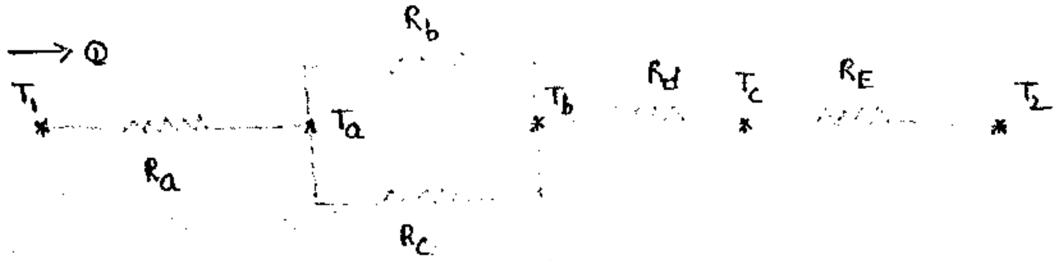
(2) Determine the heat transfer through the composite wall as shown in Fig. Take the conductivites of A, B, C, D and E as 50, 10, 6.67, 20 and 30 W/mK respectively and assume one-dimensional heat transfer



Sol:- given that

Temperature $T_1 = 800^\circ c$, $T_2 = 100^\circ c$, Area $A = 1m^2$, $k_a = 50\, W/m\text{-}k$

$k_b = 10\, W/m\text{-}k$, $k_c = 6.67\, W/m\text{-}k$, $k_d = 20\, W/m\text{-}k$, $k_E = 30\, W/mk$



heat transfer $Q = \dfrac{\Delta T_{overall}}{R_{eq}}$ where $R_{eq} \rightarrow$ equivalent resistance

$R_f \rightarrow$ Total resistance of B&C

here $R_{eq} = R_a + R_f + R_d + R_e$

$\Delta T_{overall} = T_1 - T_2 = 800^\circ c - 100^\circ c = 700^\circ c$

R_a = thermal resistance in slab A $= \dfrac{L_a}{k_a \times A} = \dfrac{5cm}{50 \times \frac{W}{mK} \times 1m^2}$

$R_a = \dfrac{5 \times 10^{-2} m}{50 \frac{W}{mK} \times 1 m^2} = 0.001 \left(W/k\right)^{-1}$

R_b = thermal resistance in slab B $= \dfrac{L_b}{k_b \times A_b} = \dfrac{10 \times 10^{-2}}{10 \times 0.5} = 0.02\, (W/K)^{-1}$

R_c = thermal resistance in slab c $= \dfrac{L_c}{k_c \times A_c} = \dfrac{10\times10^{-2}}{6.67\times(0.5)}$

$= 0.0299 = 0.03\ (W/K)^{-1}$

R_d = thermal resistance in slab d $= \dfrac{L_d}{k_d \times A_d} = \dfrac{5\times10^{-2}}{20\times1}$

$= 0.0025\ (W/K)^{-1}$

R_E = thermal resistance in slab E $= \dfrac{L_e}{k_e \times A_e} = \dfrac{5\times10^{-2}}{30\times1}$

$= 0.00166\ (W/K)^{-1}$

since B and c are parallel to each other

$$\frac{1}{R_f} = \frac{1}{R_b} + \frac{1}{R_c} = \frac{1}{0.02} + \frac{1}{0.03} = \frac{0.03+0.02}{(0.02\times0.03)} = 83.33$$

$$R_f = \frac{1}{83.33} = 0.012\ (W/K)^{-1}$$

$$R_{eq} = R_a + R_f + R_d + R_e$$

$$= 0.001 + 0.012 + 0.0025 + 0.00166 = 0.0017\ (W/K)^{-1}$$

heat transfer $\Phi = \dfrac{\Delta T_{overall}}{R_{eq}} = \dfrac{700}{0.0017} = 40768\ W$

$= 40.76\ kW$

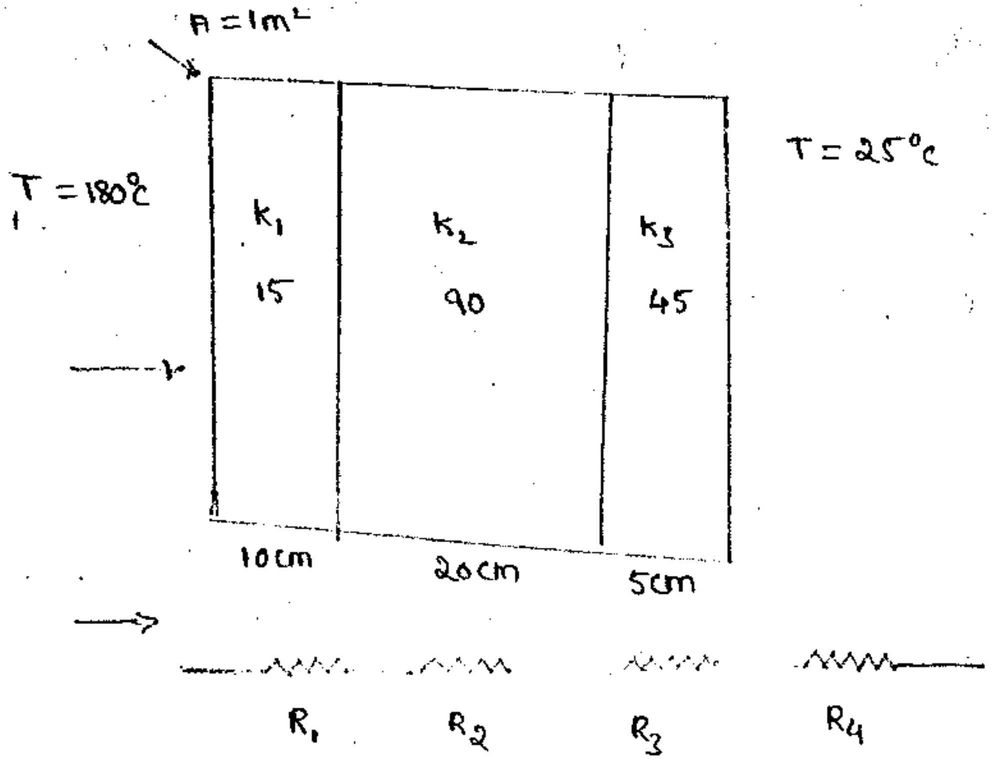
③ A furrance wall is made up of a 3 layers having conductivity $15\ W/m\text{-}k$, $90\ W/m\text{-}k$, $45\ W/m\text{-}k$ left side wall temperature $180°c$

(a) Resistance to heat flow is $0.05\ W/K$

(b) heat flow rate per unit area $3.1\ kW/m^2$.

(c) Temperature drop in second layer will be highest

(d) Temperature drop in latest in the second layer



$$R_{eq} = \frac{10\times10^{-2}}{15\times1} + \frac{20\times10^{-2}}{90\times1} + \frac{5\times10^{-2}}{45\times1} + \frac{1}{25\times1} = 0.05 \ K/W$$

$$\frac{Q}{1} = \frac{\Delta T}{\frac{R_{eq}}{1}} \Rightarrow Q = \frac{\Delta T}{R_{eq}} = \frac{(180-25)}{0.05} = 3100W = 3.1kW$$

$$Q \propto \frac{dT}{dx} \Rightarrow Q = -kA\frac{dT}{dx} \Rightarrow dT = Q\times\frac{dx}{kA}$$

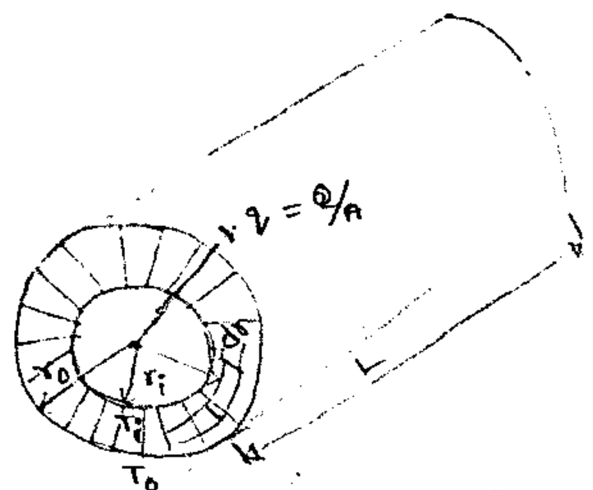
$$dT\Big|_1 = \frac{dx}{k} = \frac{10\times10^{-2}}{15} = 0.006 \ \frac{m}{W/m-K} = \frac{m^2-K}{W}$$

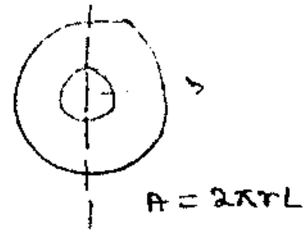
$$\frac{dx}{k}\Big|_2 = \frac{20\times10^{-2}}{90} = 0.002$$

$$\frac{dx}{k}\Big|_3 = \frac{5\times10^{-2}}{45} = 0.011$$

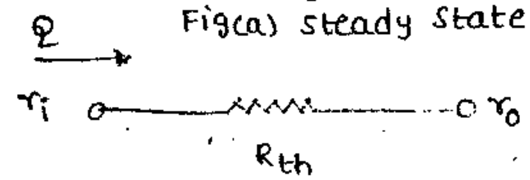
Ans) b and d

Radial heat conduction in hollow cylinders





Fig(a) steady state conduction through Hollow cylinder



According to fourier law

$$Q = -k \times A \times \frac{dT}{dr}$$

$$Q = -k \times (2\pi \times r \times L)\frac{dT}{dr}$$

The general heat conduction equation in cylindrical coordinate is

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\left[\frac{\partial T}{\partial r}\right] + \frac{1}{r^2}\left[\frac{\partial^2 T}{\partial \phi^2}\right] + \frac{\partial^2 T}{\partial z^2} + \frac{q}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial r}$$

Assuming the heat flows only in a radial direction the above under steady state (without heat generation)

$$\frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial r^2} = 0 \Rightarrow \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial}{\partial r}\left[\frac{\partial T}{\partial r}\right] = 0$$

$$\frac{\partial}{\partial r}\left[r \times \frac{\partial T}{\partial r}\right] = 0$$ Integrate the equation

$$r\frac{\partial T}{\partial r} = C_1$$ → Integrate the equation again w.r.to "r"

$$\frac{\partial T}{\partial r} = \frac{C_1}{r} \Rightarrow T = C_1 \ln r + C_2$$

Boundary conditions

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when $r = r_i \Rightarrow T = T_i \qquad r = r_o \Rightarrow T = T_o$

using boundary conditions we get

$$T_i = C_1 \ln r_i + C_2 \qquad T_o = C_1 \ln r_o + C_2$$

$$C_2 = T_i - C_1 \ln r_i \qquad T_o = C_1 \ln r_o + T_i - C_1 \ln r_i$$

$$(T_o - T_i) = C_1 (\ln r_o - \ln r_i)$$

$$C_1 = \frac{(T_o - T_i)}{\ln(r_o/r_i)}$$

w.k.t $\quad T = T_i = C_1 \ln r_i + C_2$

$$T = \left[\frac{T_o - T_i}{\ln\left(\frac{r_o}{r_i}\right)}\right] \ln r_i + \left[T_i - C_1 \ln r_i\right]$$

"T" diff w.r.to "r" we get

$$\frac{dT}{dr} = \frac{(T_o - T_i)}{\ln(r_o/r_i)} \times \frac{1}{r}$$

According fourier law

$$Q = -k \times (2\pi \times r \times L) \times \frac{dT}{dr} = -k \times 2\pi \times r \times L \times \frac{(T_o - T_i)}{\ln\left(\frac{r_o}{r_i}\right)} \times \frac{1}{r}$$

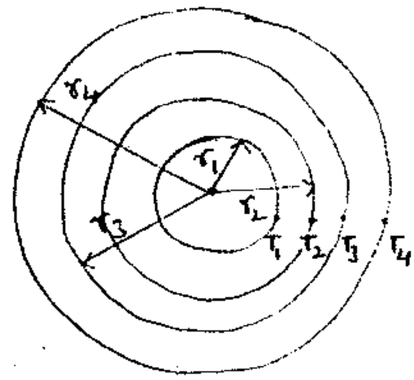
$$Q = 2\pi kL \frac{(T_i - T_o)}{\ln\left(\frac{r_o}{r_i}\right)} = \frac{(T_i - T_o)}{\left[\frac{\ln(r_o/r_i)}{2\pi kL}\right]} = \frac{\Delta T}{R_{th}}$$

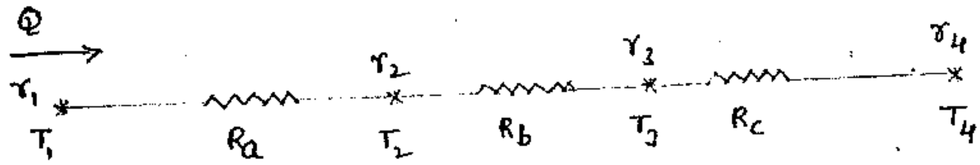
$Q = \dfrac{\Delta T}{R_{th}}$ where ΔT = overall temperature difference

R_{th} = Thermal resistance

here thermal resistance $R_{th} = \dfrac{\ln(r_o/r_i)}{2\pi kL}$

composite cylinders (or) coaxial cylinders.





W.K.T thermal resistance $R_{th} = \dfrac{\ln\left[\dfrac{r_o}{r_i}\right]}{2\pi K L}$

$$R_a = \frac{\ln\left[\dfrac{r_2}{r_1}\right]}{2\pi K_A L}, \quad R_b = \frac{\ln\left[\dfrac{r_3}{r_2}\right]}{2\pi K_B L}$$

$$R_c = \frac{\ln\left[\dfrac{r_4}{r_2}\right]}{2\pi K_B \times L}$$

If fluid "A" is passing through inside cylinder of radius r_1

$$Q = hA\Delta T = \frac{\Delta T}{(1/hA)}, \quad (1/hA) \rightarrow R_{fA}$$

$$R_{fA} = \frac{1}{h_{fa} \times 2\pi \times r_1 \times L}$$

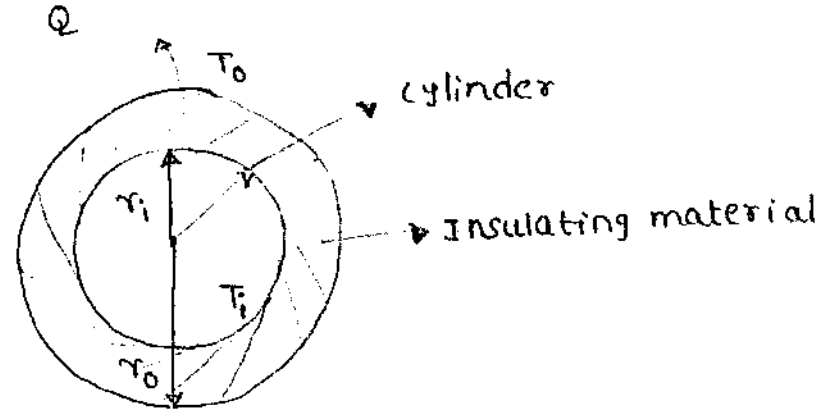
at outside of cylinder

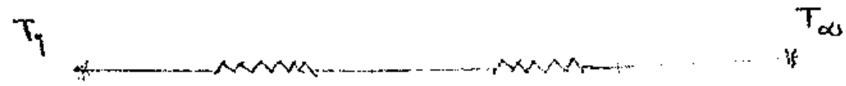
$$R_{fB} = \frac{1}{h_{fb} \times 2\pi \times r_4 \times L}$$

W.K.T heat Transfer $Q = \dfrac{\Delta T_{overall}}{R_{eq}}$

where $\Delta T_{overall} = UA\Delta T$

Critical radius of insulation





$$\frac{\ln\left(\frac{r_o}{r_i}\right)}{2\pi kL} \qquad \frac{1}{2\pi r_o L \times h}$$

$$R_{eq} = \frac{\ln\left[\frac{r_o}{r_i}\right]}{2\pi kL} + \frac{1}{2\pi \times r_o \times L \times h}$$

heat transfer

$$Q = \frac{(T_i - T_\infty)}{\frac{\ln(r_o/r_i)}{2\pi kL} + \frac{1}{2\pi r_o L h}} = \frac{2\pi L(T_i - T_\infty)}{\frac{\ln(r_o/r_i)}{k} + \frac{1}{r_o h}}$$

maximize the Q w.r.to "r_o"

$$\frac{dQ}{dr_o} = 0 \Rightarrow \frac{-2\pi L(T_i - T_\infty)\left[\frac{1}{kr_o} - \frac{1}{hr_o^2}\right]}{\left[\frac{\ln\left(\frac{r_o}{r_i}\right)}{k} + \frac{1}{r_o h}\right]^2} = 0$$

$$-2\pi L(T_i - T_\infty)\left[\frac{1}{kr_o} - \frac{1}{hr_o^2}\right] = 0$$

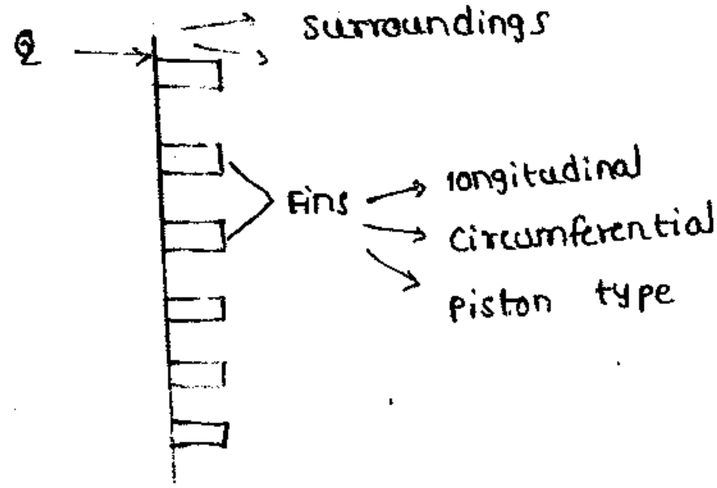
$$\frac{1}{kr_o} - \frac{1}{hr_o^2} = 0 \Rightarrow hr_o^2 = kr_o$$

$$r_o = k/h$$

UNIT-2

Extended Surface - fins

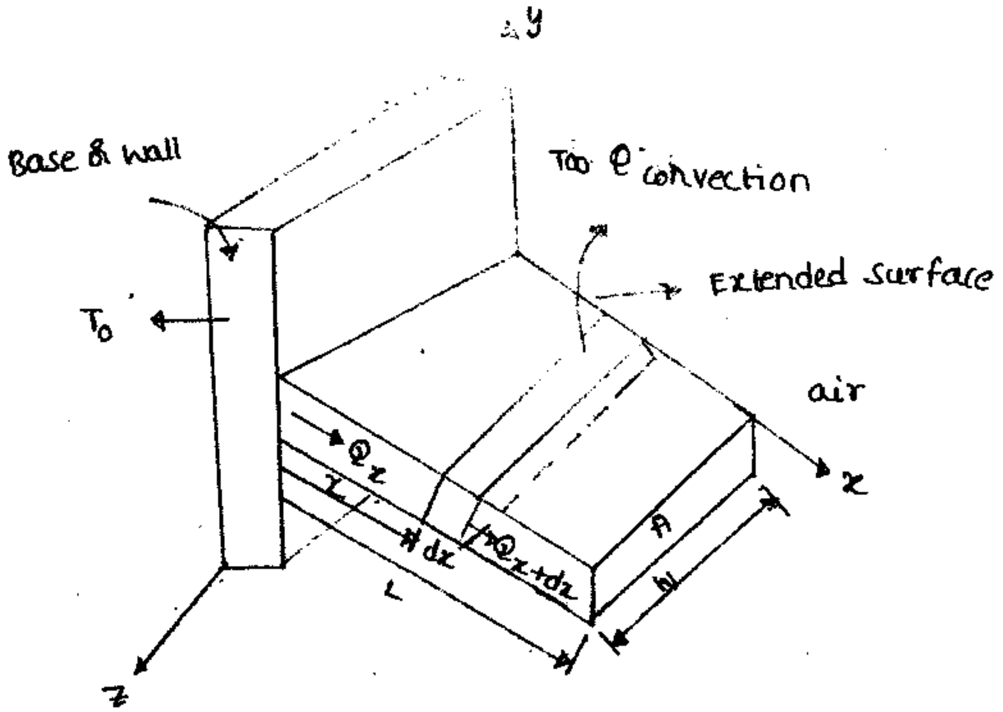
Extended surface (fins) are required to dissipate the heat inorder to increase the life of the system



Applications:-

* cooling of electric motors, electronic components, compressors, IC engines, transformer, refrigeration etc.

Rectangular fin with uniform cross-section



here heat transfer is taken place by conduction and convection

$$Q_x = Q_{x+dx} + Q_{conv}$$

$Q_x - Q_{x+dx} \longrightarrow$ Net heat conducted in the element

$$\left[Q_x - Q_{x+dx}\right] - Q_{convection} = 0$$

where

According to fourier law

$$Q_x = -kA\frac{dT}{dx} \quad , \quad Q_{x+dx} = -kA\frac{dT}{dx} + \frac{\partial}{\partial x}\left[-kA\frac{dT}{dx}\right]dx$$

Now

$$(Q_{convection})_{at\ x} = hA_s\left[T_x - T_\infty\right] \qquad \left[Q_{conv} = hA_s(T_0 - T_\infty)\right]$$

$$Q_x - Q_{x+dx} - Q_{conv} = 0$$

$$-kA\frac{dT}{dx} - \left[-kA\frac{dT}{dx} + \frac{d}{dx}\left(-kA\frac{dT}{dx}\right)dx\right] - h\left[P\times dx\right]\left[T - T_\infty\right] = 0$$

$$-kA\frac{dT}{dx} + kA\frac{dT}{dx} + kA\frac{d^2T}{dx^2}dx - h\times P\times dx\,(T - T_\infty) = 0$$

$$kA\frac{d^2T}{dx^2}dx - hP\,dx\left[T - T_\infty\right] = 0$$

$$kA\frac{d^2T}{dx^2} - hP(T - T_\infty) = 0$$

convection

net heat conduction

$$\frac{d^2T}{dx^2} - \frac{hP}{kA}(T - T_\infty) = 0$$

Let $\dfrac{hP}{kA} = m^2$

$$\frac{d^2T}{dx^2} - m^2(T - T_\infty) = 0$$

Let $(T - T_\infty) = \theta \longrightarrow$ Excess temperature

$$\frac{d^2T}{dx^2} - m^2\theta = 0$$

This equation is second order linear ordinary differential ②

equation

let us consider a trial function $\theta = e^{ax}$

Differentiate the trial function thice

$$\frac{d\theta}{dx} = a e^{ax}$$

$$\frac{d^2\theta}{dx^2} = a^2 e^{ax}$$

$$\frac{d^2\theta}{dx^2} - m^2\theta = 0 \Rightarrow a^2 e^{ax} - m^2 e^{ax} = 0$$

$$a^2 e^{ax} = m^2 e^{ax}$$

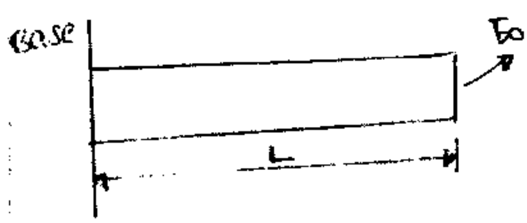
$$a = \pm m$$

$$* \; \theta = e^{ax} \Rightarrow e^{\pm mx} \Rightarrow \theta = A e^{mx} + B e^{+mx}$$

$$\theta = C_1 e^{-mx} + C_2 e^{mx}$$ where $C_1, C_2 \rightarrow$ orbitary const

long fin:-

If a fin is said to be long the temperature at the end of the fin is equal to the temperature of the surroundings



$T = T_\infty \Rightarrow$ long fin

$T \neq T_\infty \Rightarrow$ short fin

Boundary conditions $\frac{d^2\theta}{dx^2} - m^2\theta = 0$ $x \geq 0$

(where $\theta_x = T_x - T_\infty$)

at $x = 0$ $\theta = \theta_0 \Rightarrow (T_0 - T_\infty)$

$x = \infty$ $\theta = 0 \Rightarrow (T - T_\infty) = 0$

$$T = T_\infty$$

.... secondary boundary conditions

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$x = \infty \Rightarrow \theta = 0$

$$\theta = c_1 e^{-mx} + c_2 e^{mx} \Rightarrow 0 = c_1 e^{-m\infty} + c_2 e^{m\infty}$$

$$0 = c_1 \times e^{\frac{1}{m\infty}} + c_2 e^{m\infty} \Rightarrow 0 = c_1 \times 0 + c_2 \times \infty$$

when $c_2 = 0$

$$\theta = c_1 e^{-mx}$$

Applying first boundary conditions

$$x = 0, \quad \theta = \theta_0 = (T_0 - T_\infty)$$

$$\theta_0 = c_1 e^{-mx}$$

$$(T_0 - T_\infty) = c_1 e^{-mx}$$

$$(T_0 - T_\infty) = c_1 e^{-m0} \Rightarrow c_1 = (T_0 - T_\infty)$$

$$\theta = c_1 e^{-mx}$$

$$(T - T_\infty) = (T_0 - T_\infty) e^{-mx} \Rightarrow e^{-mx} = \frac{T - T_\infty}{T_0 - T_\infty}$$

heat transfer through the fin using net conduction & convection loss

$$(T - T_\infty) = \theta_0 e^{-mx}$$

$$Q = \int_0^\infty hP\,dx\,(T - T_\infty)$$

$$Q = \int_0^\infty hP\theta_0 e^{-mx}\,dx = hP\theta_0 \left.\frac{-1}{m} e^{-mx}\right]_0^\infty$$

$$Q = hP\theta_0 \times \frac{-1}{m} \times [-0-1] = \frac{1}{m} hP\theta_0 \qquad \left(m^2 = \frac{hP}{KA}\right)$$

$$Q = \frac{1}{\sqrt{\frac{hP}{KA}}} \times hP \times \theta_0$$

$$Q = \sqrt{hPKA}\,(T_0 - T_\infty)$$

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where $h \rightarrow$ convective heat transfer coefficient

$P \rightarrow$ perimeter of external surface

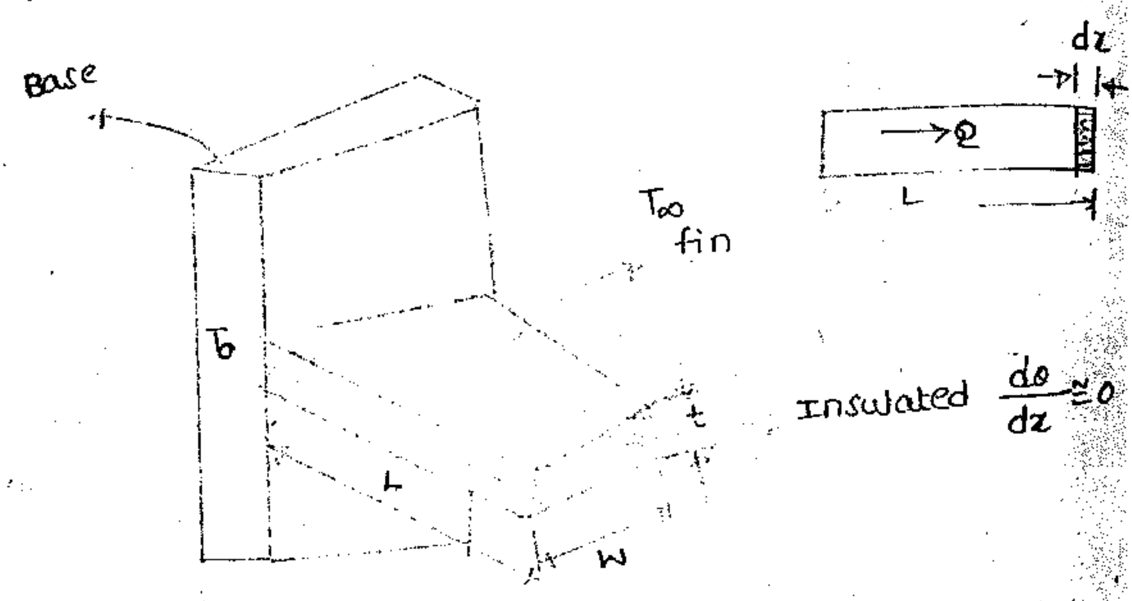
$k \rightarrow$ Thermal conductivity

$A \rightarrow$ Area perpendicular to the heat flow through conduction (cross-section area of fin)

$T_0 \rightarrow$ Base Temperature (or) wall Temperature

$T_\infty \rightarrow$ surrounding Temperature

Fin with Insulated end



General formulation for extended surfaces

$$\frac{d^2\theta}{dx^2} - m^2\theta = 0$$

Boundary conditions

at base $x = 0 \Rightarrow \theta = \theta_0 = T_0 - T_\infty$

at $x = L \Rightarrow \frac{d\theta}{dx} = 0$

Solution for General formulation $\theta = C_1 e^{-mx} + C_2 e^{mx}$

Applying boundary condition

$x = 0 \Rightarrow \theta = \theta_0$

$$\theta_0 = C_1 e^{-m(0)} + C_2 e^{m(0)}$$

$$\theta_0 = C_1 + C_2$$

Applying second boundary condition

$$\theta = C_1 e^{-mx} + C_2 e^{mx}$$

"θ" diff w.r.to "x" we get

$$\frac{d\theta}{dx} = -mC_1 e^{-mx} + mC_2 e^{mx}$$

at $x = L \rightarrow \frac{d\theta}{dx} = 0$

$$0 = -mC_1 e^{-mL} + mC_2 e^{mL}$$

$$mC_1 e^{-mL} = mC_2 e^{mL}$$

$$C_1 = \frac{C_2 e^{mL}}{e^{-mL}} = C_2 e^{2mL} \Rightarrow C_1 = C_2 e^{2mL}$$

$$\theta_0 = C_1 + C_2 = C_2 e^{2mL} + C_2 \Rightarrow \theta_0 = C_2 \left[e^{2mL} + 1\right]$$

$$C_2 = \frac{\theta_0}{(1 + e^{2mL})}$$

$$C_1 = \frac{\theta_0}{1 + e^{2mL}} \times e^{2mL}$$

$$= \frac{\theta_0}{(e^{-2mL} + 1)}$$

W.K.T $\theta = C_1 e^{-mx} + C_2 e^{mx}$

$$\theta = \frac{\theta_0}{(e^{-2mL} + 1)} e^{-mx} + \frac{\theta_0}{(e^{2mL} + 1)} \times e^{mx}$$

$$\theta = \theta_0 \left[\frac{e^{-mx}}{(e^{-2mL} + 1)} + \frac{e^{mx}}{(e^{2mL} + 1)}\right]$$

multiplying 1st term with e^{mL} and 2nd term with e^{-mL}

$$\theta = \theta_0 \left(\frac{e^{-mx} \times e^{mL}}{e^{mL}(e^{-2mL}+1)} + \frac{e^{mx} \times e^{-mL}}{e^{-mL}(e^{2mL}+1)} \right)$$

$$\theta = \theta_0 \left(\frac{e^{-m(x-L)}}{(e^{-mL}+e^{mL})} + \frac{e^{m(x-L)}}{(e^{mL}+e^{-mL})} \right)$$

$$\theta = \theta_0 \left[\frac{e^{-m(x-L)} + e^{m(x-L)}}{(e^{-mL}+e^{mL})} \right]$$

$$\left[\cosh\beta = \frac{e^{\beta}+e^{-\beta}}{2} \quad \Rightarrow \quad 2\cosh\beta = e^{\beta}+e^{-\beta} \right]$$

$$\theta = \theta_0 \left(\frac{2\cosh(m(L-x))}{2\cosh(mL)} \right]$$

$$\theta = \theta_0 \frac{\cosh[m(L-x)]}{\cosh(mL)}$$

W.K.T $\theta = T - T_\infty$

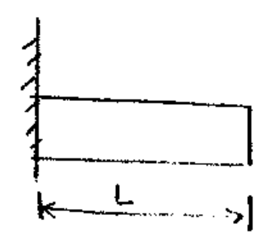
$\theta_0 = T_0 - T_\infty$

$$\frac{\theta}{\theta_0} = \frac{\cosh[m(L-x)]}{\cosh(mL)}$$

$$\frac{T-T_\infty}{T_0-T_\infty} = \frac{\cosh[m(L-x)]}{\cosh(mL)}$$

Heat transfer by the fin

$$Q = -KA \frac{d\theta}{dx}\bigg|_{x=0}$$



W.K.T $\theta = \theta_0 \dfrac{\cosh[m(L-x)]}{\cosh(mL)}$

$$\frac{d\theta}{dx} = \theta_0 \frac{\sinh[m(L-x)] \times m \times -1}{\cosh(mL)}$$

at $x=0$ $\quad Q = -KA \dfrac{d\theta}{dx}\bigg|_{x=0}$

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$$Q = -kA\theta_0 \times \frac{-m \sinh(mL)}{\cosh mL}$$

$$Q = kA\theta_0 m \tanh(mL)$$

W.K.T $m^2 = \frac{hP}{kA}$

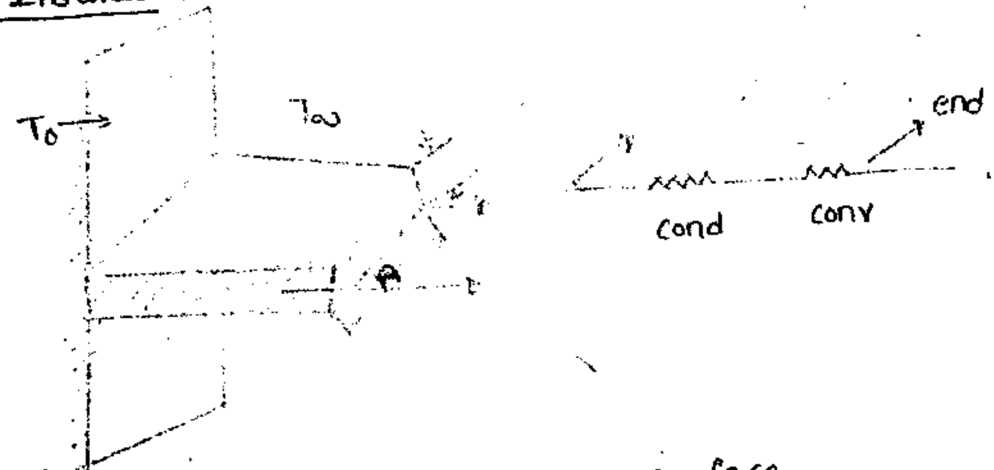
$$Q = kA \times \theta_0 \times \sqrt{\frac{hP}{kA}} \tanh(mL)$$

$$Q = \sqrt{hPkA}\ \theta_0 \tanh(mL)$$

for a long fin $\tanh(mL) \longrightarrow 1$ $\Rightarrow$ $Q = \sqrt{hPkA} \times \theta_0$

where $\theta_0 = T_0 - T_\infty$

Fins with convection of the end (Fin of finite length with end not Insulated)



General formulation for the extended surface

$$\frac{d^2\theta}{dx^2} - m^2\theta = 0 \qquad 0 \le x \le L$$

Boundary conditions

Surface Area $A_s = A$

at $x = 0$ $\quad \theta = \theta_0 = (T_0 - T_\infty)$

at $x = L$ $\quad -kA\frac{d\theta}{dx} = hA_s\theta$

$$-kA\frac{d\theta}{dx} = hA\theta \Rightarrow kA\frac{d\theta}{dx} + hA\theta = 0$$

$$k\frac{d\theta}{dx} + h\theta = 0 \rightarrow \text{at } x = L$$

W.K.T general solution

$$\theta = C_1 e^{-mx} + C_2 e^{mx}$$

at $x = 0$, $\theta = \theta_0$ $\Rightarrow$ $\theta_0 = C_1 + C_2$

at $x = L$, $\frac{k\,d\theta}{dx} = -h\theta$

$$\frac{d\theta}{dx} = -mC_1 e^{-mx} + mC_2 e^{mx}$$

$$\Rightarrow k\left[-mC_1 e^{-mx} + mC_2 e^{mx}\right] = -h\left[C_1 e^{-mx} + C_2 e^{mx}\right]$$

$$\Rightarrow m\left[-C_1 e^{-mL} + C_2 e^{mL}\right] = \frac{-h}{k}\left[C_1 e^{-mL} + C_2 e^{mL}\right]$$

W.K.T $\theta_0 = C_1 + C_2 \Rightarrow C_2 = \theta_0 - C_1$

$$m\left[-C_1 e^{-mL} + (\theta_0 - C_1) e^{mL}\right] = \frac{-h}{k}\left[C_1 e^{-mL} + (\theta_0 - C_1) e^{mL}\right]$$

$$C_1 = \frac{\theta_0\left(1 + \frac{h}{mk}\right) e^{mL}}{\left(e^{mL} + e^{-mL}\right) + \frac{h}{mk}\left(e^{mL} - e^{-mL}\right)}$$

W.K.T $$\theta = C_1 + C_2 = \frac{\theta_0\left(1 + h/mk\right) e^{mL}}{\left(e^{mL} + e^{-mL}\right) + \frac{h}{mk}\left(e^{mL} - e^{-mL}\right)} + \theta_0 - C_1$$

$$\theta = \theta_0 \frac{\cosh m(L-x) + \left(\frac{h}{mk}\right)\sinh m(L-x)}{\cosh mL + \left(\frac{h}{mk}\right)\sinh(mL)}$$

According to conduction <u>heat transfer rate</u>

$$Q = -kA\frac{d\theta}{dx}\bigg|_{x=0} = -kA \times \theta_0 \frac{-m\sinh m(L-x) + \frac{h}{mk} \times m\cosh m(L-x)}{\cosh mL + \frac{h}{mk}\sinh(mL)}$$

at $x=0 \Rightarrow Q = \dfrac{-m\sinh(mL) + \left(\frac{h}{mk}\right) m\cosh mL}{\cosh mL + \left(\frac{h}{mk}\right)\sinh(mL)} \times -kA\times\theta_0$

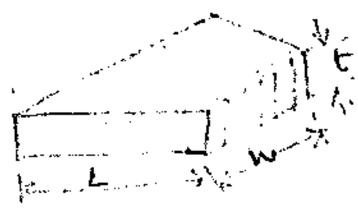
divide with $\cosh(mL)$ both numinator and denomenator

$$Q = \frac{-m\tanh(mL) + \left(\frac{h}{mk}\right)\times m\times 1}{1+\left(\frac{h}{mk}\right)\tanh(mL)} \times -kA\times\theta_0$$

$$\left[m^2 = \frac{hp}{kA}\right]$$

$$Q = \sqrt{hPkA}\,\theta_0\,\frac{\tanh(mL) + (h/mk)}{1+\left(\frac{h}{mk}\right)\tanh(mL)}$$

Rectangural fin



$A = W\times t$

perimeter $p = 2(W+t)$

$$m = \sqrt{\frac{hp}{kA}} = \sqrt{\frac{h\,2(W+t)}{k\times(W)\,t}} = \sqrt{\frac{2h(W+t)}{k\times(W)\times t}}$$

$W \gg t$

$W+t \cong W$

$$m = \sqrt{\frac{2h}{kt}} \rightarrow \text{for thin fins}$$

Pin fin (or) circular crossection fin

Area $A = \frac{\pi}{4}d^2$, $p = \pi d$

$$m = \sqrt{\frac{h\pi d}{k\frac{\pi}{4}d^2}} = \sqrt{\frac{4h}{k\times d}}$$

efficiency of the fin $\eta_{fin} = \dfrac{Q_{fin}}{Q_{max}}$

→ It is ratio of actual heat transfer by the fin to the maximum heat transferable by the fin

$$\eta_{fin} = \frac{\sqrt{hPKA} \times \theta_0}{h(PL) \times \theta_0} \rightarrow \text{long fin}$$
$\rightarrow$ convection heat transfer

<u>long fin</u> $\eta_{fin} = \frac{\sqrt{hP} \times \sqrt{KA}}{hP \times L} = \frac{\sqrt{KA}}{\sqrt{hP}} \times \frac{1}{L} = \frac{1}{mL}$

<u>efficiency f& Insulated fin</u> (rectangular fin with Insulate tip)

$$\eta_{fin} = \frac{\sqrt{hPKA}\ \theta_0 \tanh(mL)}{hPL\theta_0} = \frac{\tanh(mL)}{mL}$$

<u>Effective ness of the fin</u>

ratio of heat transfer takes place with fin and without fin

$$E = \frac{Q_{\text{with Fin}}}{Q_{\text{without fin}}} = \frac{\eta_{fin} \times A_f \times h \times \theta_0}{h \times A_b \times \theta_0}$$

$E = \frac{\sqrt{hPKA}\,(T_0 - T_\infty) \tanh mL}{hA(T_0 - T_\infty)}$

$= \frac{\tanh(mL)}{\sqrt{hP/KA}}$

$A_b \rightarrow$ Area of the base

$$E = \frac{\eta_{fin} A_f}{A_b} \geq 1$$

$= \frac{\sqrt{hPKA}\,(T_0 - T_\infty)}{hA(T_0 - T_\infty)}$ $A_f \rightarrow$ Area of the fin

rectangular $E = \sqrt{\frac{PK}{hA}}$

① one end of a very long aluminium rod is connected to a wall at 140°c, while the other end protrudes into a room whose air temperature is 15°c. The rod is 3mm in diameter and the heat transfer coefficient between the rod surface and environment is 300 W/m²k. Estimate the total heat dissipated by the rod taking its thermal conductivity as 150 W/mk

Sol: given

Base temperature & wall temperature $T_0 = 140°c$

surrounding Temperature $T_\infty = 15°c$

diameter of rod $d = 3mm = 3 \times 10^{-3} m$

heat transfer coefficient b/w rod surface and environment

$h = 300\ W/m^2\text{-}k$, thermal conductivity of rod $k = 150\ W/m\text{-}k$

since the rod is very long

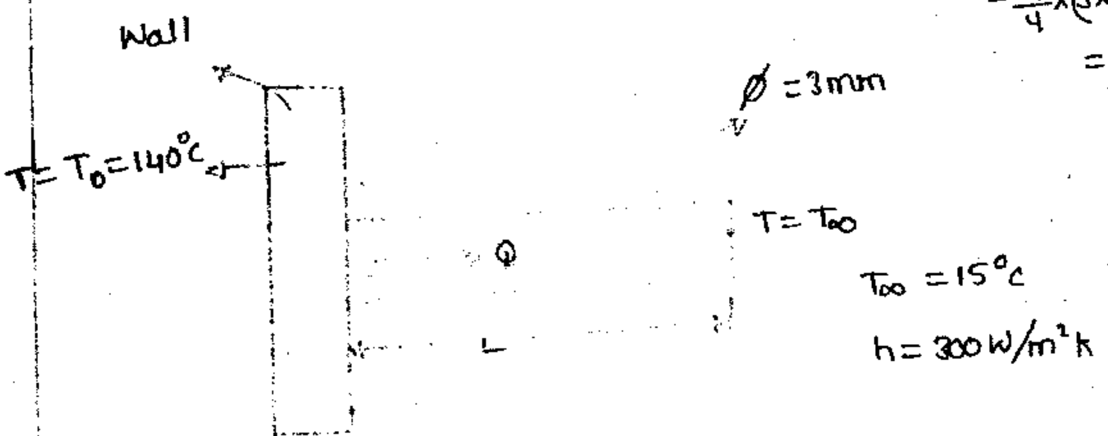
$$Q = \sqrt{hPkA}\,(T_0 - T_\infty)$$

$$Q = \sqrt{(300)(9.424\times10^{-3})\,150\,(7.068\times10^{-6})} \times (140-15)$$

$$Q = 0.0547 \times 125 = 6.843\ W$$

[P → perimeter of external surfaces $P = \pi D = 2\pi R$
$= \pi \times 3\times10^{-3}$
$= 9.424\times10^{-3}\ m$

A → cross-section Area of fin $A = \frac{\pi}{4} d^2$
$= \frac{\pi}{4}\times(3\times10^{-3})^2$
$= 7.068\times10^{-6}\ m^2$]



∴ The total heat dissipated by the rod $Q = 6.843\ W$

② A turbine blade 6cm long and having a cross-sectional area $4.65 cm^2$ and perimeter 12cm, is made of stainless steel ($K = 23.3\ W/mk$). The temperature at the root is 500°c. The blade is exposed to a hot gas at 870°c. The heat transfer coefficient between the blade surfaces and gas is $442\ W/m^2\text{-}k$. Determine the temperature distribution and rate of heat flow at the root of the blade. Assume the tip of the blade to be insulated.

given that

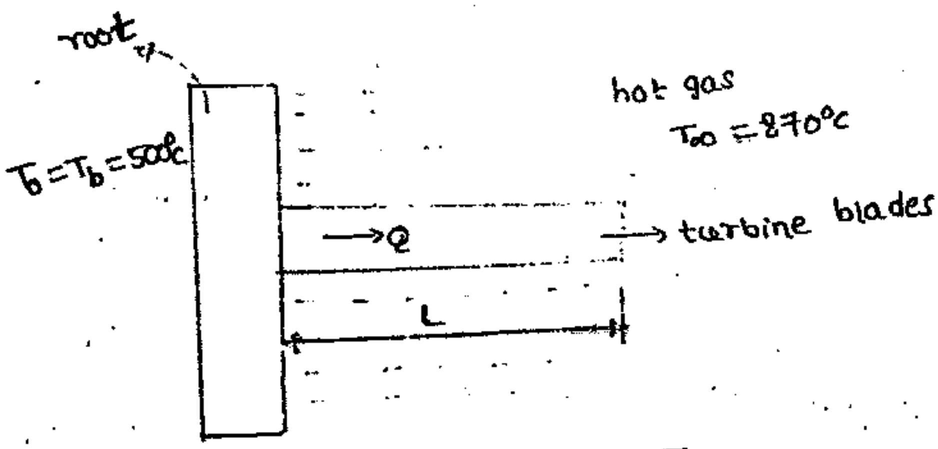
turbine blade length $L = 6cm = 6\times10^{-2} m$

cross-sectional Area $A = 4.65 cm^2$, perimeter $P = 12cm = 12\times10^{-2} m$, thermal conductivity $k = 23.3 W/mK$, temperature at surrounding $T_\infty = 870°c$, Base temperature $T_0 = 500°c$, heat transfer coefficient between the blade surface and gas

$h = 442 W/m^2k$

This is the case of a fin with an Insulated end, for which we can use equation for temperature distribution

$$\frac{T - T_\infty}{T_0 - T_\infty} = \frac{\cosh[m(L-x)]}{\cosh(mL)}$$



$$m = \sqrt{\frac{hP}{kA}} = \sqrt{\frac{442\times12\times10^{-2}}{4.65\times10^{-4}\times23.3}} = 69.96$$

$$\text{W.K.T} \quad T - T_\infty = (500-870°)\frac{\cosh[69.96(6\times10^{-2}-x)]}{\cosh[69.96(6\times10^{-2})]}$$

$$= -370\frac{\cosh[69.96(6\times10^{-2}-x)]}{\cosh[69.96\times6\times10^{-2}]}$$

$$T - T_\infty = -11.12\cosh[69.96(6\times10^{-2}-x)]$$

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The heat transfer rate is given by eqn

$$Q = \sqrt{hPKA}\,\theta_0 \tanh mL$$

$$= \sqrt{442 \times 12\times10^{-2} \times 23.3 \times 4.65\times10^{-4}}\,(500-870) \tanh(69.96 \times 6\times10^{-2})$$

$$= 0.758 \times -370 \times 0.9995$$

$$Q = -280.3\,W$$

③ compare the temperature distributions in a spine (pin fin) having a diameter of 2cm and length 10cm and exposed to a convection environment with $h = 25\,W/m^2K$ for three fin materials copper ($k = 385\,W/mK$), stainless steel ($k = 17\,W/mK$) and glass $k = 0.8\,W/mK$. Also compare the relative heat flows and fin efficiencies with respect to the copper fin

Sol: given that

Assume fin is short and insulated at end

diameter $d = 2cm = 2\times10^{-2}m$, length of short fin $L = 10cm = 10\times10^{-2}m$

$h = 25\,W/m^2K$, thermal conductivity of three fin materials

copper $k = 385\,W/mK$, stainless steel $k = 17\,W/mK$, glass $k = 0.8\,W/mK$

W.K.T

$$\frac{T_x - T_\infty}{T_0 - T_\infty} = \frac{\cosh[m(L-x)]}{\cosh(mL)}$$

at $x = L$, $$\frac{T_L - T_\infty}{T_0 - T_\infty} = \frac{\cosh[m(L-L)]}{\cosh(mL)} = \frac{\cosh(0)}{\cosh(mL)} = \frac{1}{\cosh(mL)}$$

for copper $$m = \sqrt{\frac{hP}{kA}} = \sqrt{\frac{25 \times [2\pi \times 1\times10^{-2}]}{385 \times \frac{\pi}{4} \times (2\times10^{-2})^2}} = \sqrt{\frac{4\times25}{385\times2\times10^{-2}}}$$

$$m = 3.60$$

$$\frac{T_L - T_\infty}{T_0 - T_\infty} = \frac{1}{\cosh\left[3.60 \times 10 \times 10^{-2}\right]} = 0.9385$$

$$(T_0 - T_\infty) = \theta_0 \quad \Rightarrow \quad T_L - T_\infty = 0.9385\,\theta_0 = 0.9385\,(T_0 - T_\infty)$$

$$\theta_0 = \frac{T_L - T_\infty}{0.9385}$$

for stainless steel

at $x = L$

$$\frac{T_L - T_\infty}{T_0 - T_\infty} = \frac{\cosh[m(L-x)]}{\cosh(mL)} = \frac{\cosh[m(L-L)]}{\cosh(mL)}$$

$$= \frac{\cosh(0)}{\cosh(mL)} = \frac{1}{\cosh(mL)}$$

$$m = \sqrt{\frac{hP}{kA}} = \sqrt{\frac{25 \times (2 \times \pi \times 1 \times 10^{-2})}{17 \times \frac{\pi}{4} \times (2 \times 10^{-2})^2}} = \sqrt{\frac{4 \times 25}{17 \times 2 \times 10^{-2}}}$$

$$m = 17.1$$

$$\frac{T_L - T_\infty}{T_0 - T_\infty} = \frac{1}{\cosh\left[17.1 \times 10 \times 10^{-2}\right]} = 0.3486$$

$$T_L - T_\infty = 0.3486 \times \theta_0 = 0.3486\,(T_0 - T_\infty)$$

$$\theta_0 = \frac{T_L - T_\infty}{0.3486}$$

for glass

at $x = L$

$$\frac{T_L - T_\infty}{T_0 - T_\infty} = \frac{1}{\cosh(mL)}$$

$$m = \sqrt{\frac{hP}{kA}} = \sqrt{\frac{25 \times (2 \times \pi \times 1 \times 10^{-2})}{0.8 \times \frac{\pi}{4} \times (2 \times 10^{-2})^2}} = \sqrt{\frac{25 \times 4}{0.8 \times 2 \times 10^{-2}}} = 79.05$$

$$\frac{T_L - T_\infty}{T_0 - T_\infty} = \frac{1}{\cosh\left[79.05 \times 10 \times 10^{-2}\right]} = 0.00073$$

$$T_L - T_\infty = 0.00073\,\theta_0 = 0.00073\,(T_0 - T_\infty) \Rightarrow \theta_0 = \frac{T_L - T_\infty}{0.00073}$$

heat flow for a copper fin

$$Q = \sqrt{hPkA} \times \theta_0 \times \tanh(mL)$$

$$Q = \sqrt{25\times(2\pi\times1\times10^{-2})\times385\times\frac{\pi}{4}\times(2\times10^{-2})^2} \times \frac{T_L - T_\infty}{0.9385} \times \tanh[(3.60\times10\times10^{-2})]$$

$$Q_{copper} = 0.1603\,(T_L - T_\infty)$$

$$Q_{\text{stainless steel}} = \sqrt{25\times(2\pi\times10^{-2})\times17\times\frac{\pi}{4}\times(2\times10^{-2})^2} \times \frac{T_L - T_\infty}{0.2486} \times \tanh[(17.1\times10\times10^{-2})]$$

$$Q_{\text{stainless steel}} = 0.2462\,(T_L - T_\infty)$$

$$Q_{glass} = \sqrt{25\times(2\pi\times1\times10^{-2})\times0.8\times\frac{\pi}{4}\times(2\times10^{-2})^2} \times \frac{T_L - T_\infty}{0.00073} \times \tanh[(79.05\times10\times10^{-2})]$$

$$Q_{glass} = 26.9\,(T_L - T_\infty)$$

compare the efficiencies with respect to the copper

$$\eta_{ss} = \frac{0.2462(T_L - T_\infty)}{0.1603\,(T_L - T_\infty)} = 1.53$$

short fin

$$\text{efficiency of copper fin} = \eta_{fin} = \frac{Q_{fin}}{Q_{max}} = \frac{\sqrt{hPkA}\,\theta_0\tanh(mL)}{(hA'\Delta T) = hPL\,\theta_0}$$

$$\eta_{fin} = \frac{\tanh(mL)}{mL} = \frac{\tanh(0.36)}{0.36}$$

$$mL_{copper} = 3.60\times10\times10^{-2} = 0.36$$

$$\eta_{fin_{copper}} = 0.958 = 95.8\,\%$$

for stainless steel

$$\eta_{fin} = \frac{\tanh(mL)}{mL} \Rightarrow \eta_{fin} = \frac{\tanh(1.71)}{1.71}$$

$mL = 17.1 \times 10 \times 10^{-2} = 1.71$

$= 0.547 = 54.7\%$

for glass

$$\eta_{fin} = \frac{\tanh(mL)}{mL} \Rightarrow \eta_{fin} = \frac{\tanh(7.905)}{7.905}$$

$mL = 79.05 \times 10 \times 10^{-2} = 7.905$

$= 0.126 = 12.6\%$

comparing copper with copper $\Rightarrow \eta_{fin} = 100\%$

comparing stainless stell with copper $\Rightarrow \eta_{fin} = 52.4\%$

comparing glass with copper $\Rightarrow \eta_{fin} = 12.07\%$

Transient heat conduction

* Temperature is function of variables (x, y, z, t) steady state $\frac{\partial T}{\partial t} = 0$

Transient unsteady $\frac{\partial T}{\partial t} \neq 0$

- periodic heat transfer
- non - periodic heat transfer

Periodic heat transfer

The temperature varies on regular basis (or) The particular instant of time Temperature remains constant is called is periodic heat transfer non - periodic heat transfer

The temperature at any point varies non-linearly with the time.

Applications :

① Boiler tubes ② Automobile enginess ③ Rocket nozzles ④ cooling and freezing of food ⑤ preservation of medicines ⑥ heating and cooling of buildings ⑦ heat treatment of metals

Lumped heat analysis (or) systems with Negligible Internal resistance

① The temperature distribution across a large concrete slab 50cm thick heated from one side as measured by thermocouples approximates to the following relation $T = 60 - 50x + 12x^2 + 20x^3 - 15x^4$ where T is in °c and x is in meters considering an area of 5m², compute (i) the heat entering and leaving the slab in unit time (ii) The heat energy stored in unit time (iii) The rate of temperature change at both sides of the slab (iv) the point where the rate of heating or cooling is maximum. Take the following data for concrete $k = 1.2 W/mK$, $\alpha = 1.77 \times 10^{-3} m^2/hr$

Sol:- given that

$$T = 60 - 50x + 12x^2 + 20x^3 - 15x^4, \quad k = 1.2 W/mK$$

$$\alpha = 1.77 \times 10^{-3} m^2/hr$$

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{q'}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

→ It is one-dimensional problem, unsteady state

* no heat internal generation

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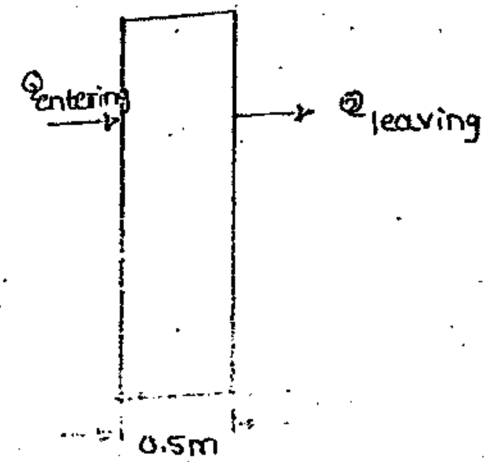
$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

According to fourier law, here heat entering and leaving the system is depend on temperature gradient $\left(\frac{\partial T}{\partial x}\right)$

$$Q_{entering} = -kA\left[\frac{\partial T}{\partial x}\right]_{x=0}$$

$Q_{net} = Q_{entering} - Q_{leaving}$

$$Q_{leaving} = -kA\left[\frac{\partial T}{\partial x}\right]_{x=0.5}$$



$$T = 60 - 50x + 12x^2 + 20x^3 - 15x^4$$

$$\frac{\partial T}{\partial x} = 0 - 50 + 24x + 60x^2 - 60x^3$$

$$\left[\frac{\partial T}{\partial x}\right]_{x=0} \Rightarrow \frac{\partial T}{\partial x} = -50$$

$$\left[\frac{\partial T}{\partial x}\right]_{x=0.5} \Rightarrow \frac{\partial T}{\partial x} = -50 + 24(0.5) + 60(0.5^2) - 60(0.5^3)$$

$$\frac{\partial T}{\partial x} = -30.5$$

(i) $Q_{entering} = -kA\left[\frac{\partial T}{\partial x}\right]_{x=0} = [-1.2 \times 5] \times [-50] = 300W$

$Q_{leaving} = -kA\left[\frac{\partial T}{\partial x}\right]_{x=0.5} = [-1.2 \times 5] \times [-30.5] = 183W$

(ii) heat stored in unit time

$Q_{net} = Q_{Entering} - Q_{leaving} = 300 - 183 = 117W$

(iii) The rate of temperature change at both sides of the slab

$\left[\frac{\partial T}{\partial t}\right]_{x=0}$ and $\left[\frac{\partial T}{\partial t}\right]_{x=0.5}$

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for 1-D

$$\left[\frac{\partial^2 T}{\partial x^2}\right]_{x=0} = \frac{1}{\alpha}\left[\frac{\partial T}{\partial t}\right]_{x=0} \Rightarrow \left.\frac{\partial T}{\partial t}\right|_{x=0} = \alpha\left(\frac{\partial^2 T}{\partial x^2}\right)$$

$$\frac{\partial T}{\partial x} = -50 + 24x + 60x^2 - 60x^3$$

$$\frac{\partial^2 T}{\partial x^2} = 0 + 24 + 120x - 180x^2$$

$$\left.\frac{\partial^2 T}{\partial x^2}\right|_{x=0} = 24 \quad , \quad \left.\frac{\partial^2 T}{\partial x^2}\right|_{x=0.5} = 24 + 120(0.5) - 180(0.5^2)$$

$$= 39$$

$$\left.\frac{\partial T}{\partial t}\right|_{x=0} = \alpha \left.\frac{\partial^2 T}{\partial x^2}\right|_{x=0} = 1.77 \times 10^{-3}\, m^2/hr \times 24^\circ C$$

$$= 0.0428\,^\circ C/hr$$

$$\left.\frac{\partial T}{\partial t}\right|_{x=0.5} \Rightarrow \alpha \left.\frac{\partial^2 T}{\partial x^2}\right|_{x=0.5} = 1.77 \times 10^{-3} \times 39 = 0.069\,^\circ C/hr$$

(iv) For the rate of heating & cooling to be maximum

$$\frac{\partial}{\partial x}\left[\frac{\partial T}{\partial t}\right] = 0$$

$$\frac{\partial}{\partial x}\left[\alpha \times \frac{\partial^2 T}{\partial x^2}\right] = 0 \Rightarrow \alpha \frac{\partial^3 T}{\partial x^3} = 0$$

$$\frac{\partial^3 T}{\partial x^3} = 0$$

W.K.T

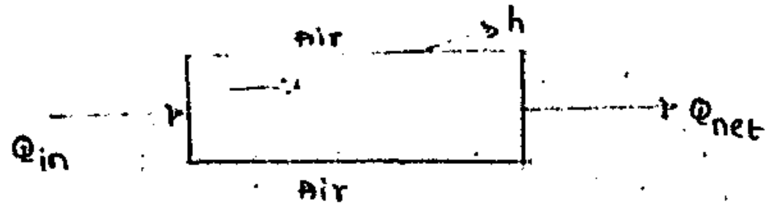
$$\frac{\partial^2 T}{\partial x^2} = 24 + 120x - 180x^2$$

$$\frac{\partial^3 T}{\partial x^3} = 0 + 120 - 360x = 0 \Rightarrow 360x = 120$$

$$x = 0.33\,m$$

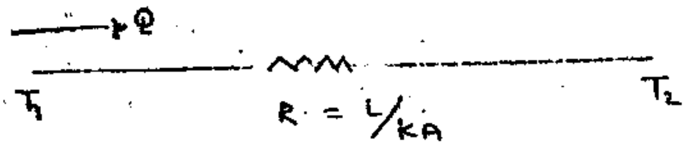
LUMPED heat analysis & systems with negligible Internal resistance

→ heat transfer in heating & cooling of a body is depend upon both the internal and surface resistance



[Internal resistance $= 1/kA$

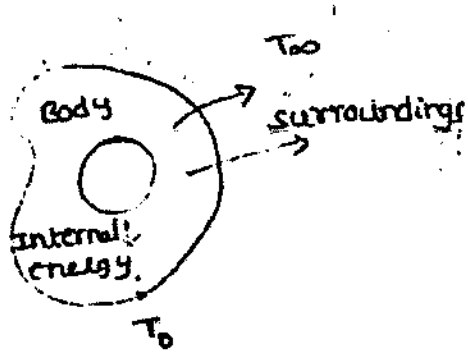
Surface resistance $= 1/hA$]



To analyse unsteady state problem It is easy by neglecting internal resistance, because surface resistance due to convection is large compare to internal resistance due to conduction

Lumped heat analysis : In a newtonian heating & cooling process the temperature throughout the solid is considered to be uniform at a given time

* Energy is a function of its temperature and total heat capacity at a given time → one lump



Lumped heat capacity of the body $Q = \rho c V$

$\rho \rightarrow$ density of the material

$c \rightarrow$ specific heat capacity

$V \rightarrow$ Volume of the body

$\rho \times V \rightarrow$ mass

$$Q = \rho c V \frac{dT}{dt} = -hA(T - T_\infty)$$

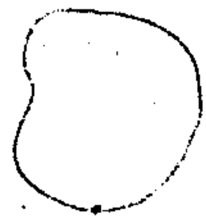
$$\frac{dT}{dt} = \frac{-hA}{\rho c V}(T - T_\infty)$$

T → Temperature at any instant of time

$$\frac{dT}{(T - T_\infty)} = \frac{-hA}{\rho c V}\, dt \longrightarrow (1)$$

By integrating equation (1)

$$\ln(T - T_\infty) = \frac{-hA}{\rho c V} t + C_1$$



$T_0 = 0,\ t = 0$

Boundary conditions

at $t = 0$, $T = T_0$

$$\ln(T_0 - T_\infty) = \frac{-hA}{\rho c V} \times 0 + C_1$$

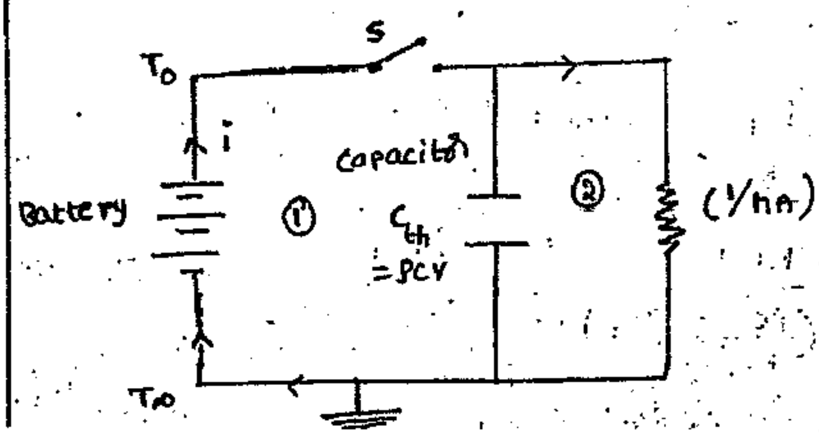
$$C_1 = \ln(T_0 - T_\infty)$$

$$\ln(T - T_\infty) = \frac{-hA}{\rho c V} t + \ln(T_0 - T_\infty)$$

$$\ln(T - T_\infty) - \ln(T_0 - T_\infty) = \frac{-hA}{\rho c V} t$$

$$\ln\left[\frac{T - T_\infty}{T_0 - T_\infty}\right] = \frac{-hA}{\rho c V} t$$

$$\frac{T - T_\infty}{T_0 - T_\infty} = e^{\left[\frac{-hA}{\rho c V} t\right]} \longrightarrow \text{Temperature distribution equation}$$



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Thermal $Q = (\rho C V) T = C_{th} T$

let us consider charge stored in the capacitor "S", V is the voltage, C is the capacitance

Electrical $S = C \times V$, w.r.t $Q = hA\Delta T = \frac{\Delta T}{(1/hA)} = \frac{\Delta T}{R_{th}}$

$$\frac{hA}{\rho CV} = \frac{1}{R_{th} C_{th}} = \frac{1}{R_e C_e}$$

where $R_{th} \rightarrow$ thermal resistance, $C_{th} \rightarrow$ thermal capacity

$R_e \rightarrow$ electrical resistance, $C_e \rightarrow$ electrical capacitance

Biot number $\longrightarrow$ It is a non-dimensional parameter to test the validity of the lumped heat capacity approach

$$B_i = \frac{\text{Internal resistance}}{\text{convective resistance}} = \frac{L/kA}{1/hA} = \frac{hL_c}{k}$$

where $L_c \longrightarrow$ characteristic length of the body

$B_i < 0.1$, for simple shapes are plates, cylinders, spheres and cubes

$$\left(\frac{T - T_\infty}{T_0 - T_\infty}\right) = e^{\left[-\frac{hA}{\rho CV} \times t\right]}$$

$$\frac{hAt}{\rho CV} = \frac{hAt}{\rho CAL_c} = \frac{ht}{\rho CL_c} \times \frac{L_c}{L_c} \times \frac{k}{k} = \frac{hL_c t}{\rho C L_c^2} \times \frac{k}{k}$$

$$\frac{1}{\alpha} = \frac{\rho C_p}{k} \longrightarrow \text{thermal diffusivity}$$

$$\frac{hAt}{\rho CV} = \frac{hL_c t}{\left(\frac{\rho C}{k} \times L_c^2 k\right)} = \frac{hL_c t}{1/\alpha \times L_c^2 \times k} = \frac{hL_c}{k} \times \frac{\alpha t}{L_c^2}$$

$$\frac{hAt}{\rho c V} = \left[\frac{hL_c}{k}\right] \times \left[\frac{\alpha t}{L_c^2}\right] = B_i \times F_o$$

where

Biot number $B_i = \frac{hL_c}{k}$, fourier number $F_o = \frac{\alpha t}{L_c^2}$

$$\left(\frac{T-T_\infty}{T_0-T_\infty}\right) = e^{\left[\frac{-hA}{\rho c V} t\right]} = e^{\left[-B_i \times F_o\right]}$$

The heat rate Q at any time t can be obtained from eq.

$$Q = \rho c V \frac{dT}{dt} = -hA\,(T-T_\infty) \Rightarrow \frac{dT}{dt} = \frac{-hA}{\rho c V}(T-T_\infty)$$

$$Q = \rho c V \frac{dT}{dt} = \rho c V \times \frac{-hA}{\rho c V}(T-T_\infty)$$

total quantity of heat during time interval $(0, t)$ is

$$U = \int_0^t Q\, dt$$

here $T - T_\infty = \frac{-\rho c V}{hA}\left(\frac{dT}{dt}\right)$

$$\frac{T-T_\infty}{T_0-T_\infty} = e^{\left[\frac{-hA}{\rho c V} \times t\right]}$$

$$(T-T_\infty) = e^{\left[\frac{-hA}{\rho c V} t\right]} \times (T_0-T_\infty)$$

$$(T_0-T_\infty)\, e^{\left[\frac{-hA}{\rho c V} t\right]} = -\frac{\rho c V}{hA}\left[\frac{dT}{dt}\right]$$

$$\frac{dT}{dt} = -\left[\frac{hA}{\rho c V}\right](T_0-T_\infty)\, e^{\left(\frac{-hA}{\rho c V} \times t\right]}$$

$$Q = \rho c V \frac{dT}{dt} = \rho c V \times \left[\frac{-hA}{\rho c V}\right] \times (T_0 - T_\infty) \times e^{\left[\frac{-hA}{\rho c V} \times t\right]}$$

Total quantity of heat given during time interval (0, t)

$$U = \int_0^t Q\,dt = \int_0^t \left[-hA\,(T_0 - T_\infty)\, e^{\left[\frac{-hA}{\rho c V} \times t\right]}\right] dt$$

$$U = -hA\,(T_0 - T_\infty) \frac{1}{\left[\frac{-hA}{\rho c V}\right]} e^{\left[\frac{-hA}{\rho c V} \times t\right]} \Big]_0^t$$

$$U = -hA\,(T_0 - T_\infty) \frac{1}{\frac{-hA}{\rho c V}} \left[e^{\left(\frac{-hA}{\rho c V}\right)t} - 1 \right]$$

$$U = \rho c V\,(T_0 - T_\infty) \left[e^{\left[\frac{-hA}{\rho c V}\right]t} - 1 \right]$$

$$U = \rho c V\,(T_0 - T_\infty) \left[e^{(-B_i \times F_0)} - 1 \right]$$

① A 40×40cm copper slab 5mm thick at a uniform temperature of 250°c suddenly has its surface temperature lowered to 30°c. Find the time at which the slab temperature becomes 90°c, density = 9000 kg/m³, c = 0.38 KJ/kg k, k = 370 W/m k, h = 90 W/m²-k

Sol:- given that

Area of copper slab = 40×40 cm × 2 = 2×0.4×0.4 = 0.32 m² (two sides)

thickness of slab t = 5mm = 0.005m

Base temperature $T_0 = 250°c$, surrounding temperature $T_\infty = 30°c$, slab temperature $T = 90°c$, density $\rho = 9\times10^3$ kg/m³, specific heat capacity c = 0.38 KJ/kg-k = 380 J/kg-k

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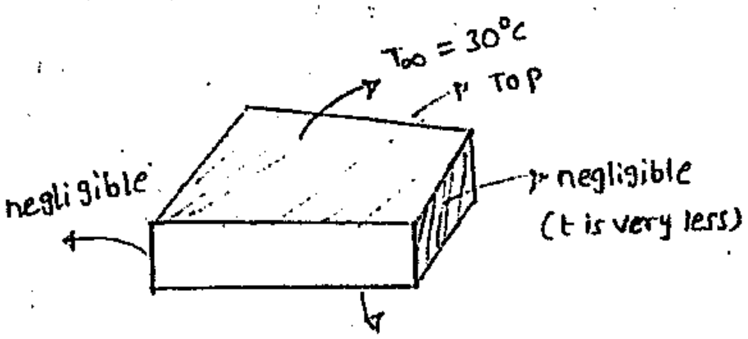
thermal conductivity $k = 370\ W/mK$, convective heat transfer coefficient $h = 90\ W/m^2K$



The general temperature distribution equation is

$$\frac{T - T_\infty}{T_0 - T_\infty} = e^{\left[\left(\frac{-hA}{\rho C V}\right) \times t\right]} = e^{(Bi \times F_0)}$$

time

$$\frac{90-30}{250-30} = e^{\left[\left(\frac{-90 \times A}{9 \times 10^3 \times 0.38 \times 10^3 \times V}\right) \times t\right]}$$

W.K.T volume $V = 0.4 \times 0.4 \times 0.005 = 8 \times 10^{-4} m^3$

Characteristic length $L_c = \frac{V}{A} = \frac{8 \times 10^{-4}}{0.32} = 2.5 \times 10^{-3} m = L$

Biot number $B_i = \frac{hL_c}{k} = \frac{90 \times 2.5 \times 10^{-3}}{370} = 6.1 \times 10^{-4} < 0.1$

here $\frac{-hA}{\rho C V} = \frac{-90 \times 0.32}{9000 \times 0.38 \times 10^3 \times 8 \times 10^{-4}} = -0.0105t$

$$\frac{90-30}{250-30} = e^{[-0.0105t]} \Rightarrow 0.273 = e^{(-0.0105t)}$$

$$\ln(0.273) = -0.0105t$$

$$-1.298 = -0.0105t$$

$$t = 123.69\ s.$$

(14)

(2) A stainless steel rod of outer diameter 1cm originally at a temperature of 320°c is suddenly immersed in a liquid at 120°c for which the convective heat transfer coefficient is 100 W/m²-k. Determine the time required for the rod to reach a temperature of 200°c

Sol: given that

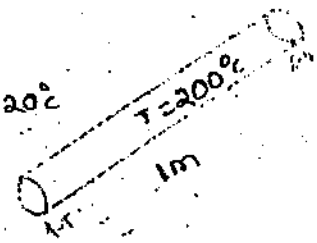
outer diameter of stainless steel rod $D = 1cm = 0.01m$

Base temperature $T_0 = 320°c$, Surrounding & operating temperature $T_\infty = 120°c$, Convective heat transfer coefficient $h = 100\ W/m^2k$

temperature of rod $T = 200°c$

W.K.T Biot number $B_i = \frac{hL_c}{K}$

$T_\infty = 120°c$



Taking 1meter length of wire

volume of rod $= V = \frac{\pi}{4} D^2 \times L = \frac{\pi}{4} \times 0.01^2 \times 1 = 7.85 \times 10^{-5} m^3$

Area of rod under heat transfer $A = \pi D L = \pi \times 0.01 \times 1$
$= 0.0314 m^2$

W.K.T $V = A \times L_c \Rightarrow L_c = \frac{V}{A} = \frac{\frac{\pi}{4} D^2 \times L}{\pi D L} = \frac{D}{4} = \frac{0.01}{4}$

$L_c = 2.5 \times 10^{-3} m$

since $B_i = \frac{hL_c}{k} = \frac{100 \times 2.5 \times 10^{-3}}{40} = 6.25 \times 10^{-3} << 0.1$

Assume for stainless steel

$\rho = 7800\ kg/m^3$, $c = 460\ J/kgK$, $k = 40\ W/mK$

the lumped capacity analysis is applicable. It follows that

$$\frac{T - T_\infty}{T_0 - T_\infty} = e^{\left[\left(\frac{-hA}{\rho c V}\right) t\right]}$$

$$\left[\frac{A}{V} = \frac{1}{D/4} = \frac{4}{D}\right]$$

W.K.T $\frac{hA}{\rho c V} = \frac{h \times A}{\rho c \times V} = \frac{4h}{\rho c D} = \frac{4 \times 100}{7800 \times 460 \times 0.01}$

$$= 0.01115/s$$

$$\frac{200 - 120}{320 - 120} = e^{[-0.0115 t]}$$

$$0.4 = e^{[-0.0115t]} \Rightarrow -0.9162 = -0.01115t$$

$$t = 82.17 \text{ sec}$$

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(15)

(2) Transient heat conduction problem

In quenching process a copper plate of 3mm thickness is heated up to 350°c and is suddenly dipped into water bath and cooled to 25°c calculate the time required for the plate to reach the temperature of 50°c. The heat transfer coefficient on the surface of the plate is 28 W/m²-k. The length and width of the plates are 40cm and 30cm respectively. The properties of copper are as follows: specific heat = 380.9 J/kg-k, density = 8800 kg/m³ and thermal conductivity = 385 W/m-k

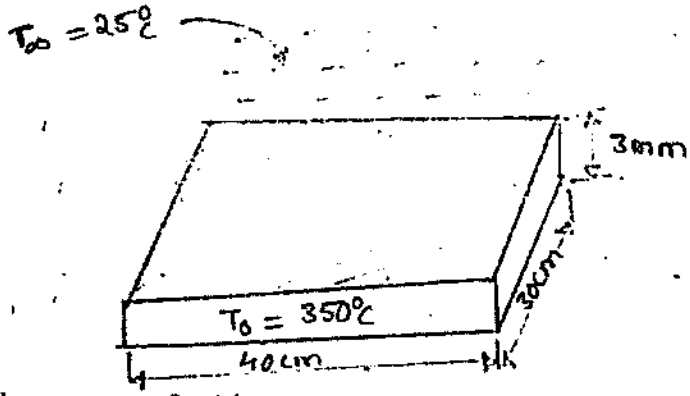
sol:-

given that

$T_0 = 350°c$, $T_\infty = 25°c$, $T = 50°c$, Area $A = [40\times10^{-2}\times30\times10^{-2}]\times2$

$= 0.24 m^2$, volume $V = L\times S\times h = 40\times10^{-2}\times30\times10^{-2}\times3\times10^{-3} m^3 = 0.00036 m^3$

Biot number $(B_i) = \frac{hL_c}{K}$



$$B_i = \frac{hL_c}{k} = \frac{28\times L_c}{385}$$

$$L_c = \frac{V}{A} = \frac{0.00036}{0.24} = 0.0015 m$$

$$B_i = \frac{28\times0.0015}{385} = 0.000109 < 0.1$$

$B_i < 0.1 \rightarrow$ Lumped heat analysis is applicable

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$$\frac{T - T_\infty}{T_0 - T_\infty} = e^{\left[\frac{-hA}{\rho c V} \times t\right]}$$

$$\frac{50 - 25}{350 - 25} = e^{\left[\frac{-hA}{\rho c V} \times t\right]}$$

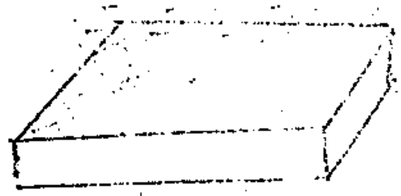
$$\frac{hA}{\rho c V} = \left[\frac{28 \times 0.24}{8800 \times 380.9 \times 0.0036}\right] = 0.0056$$

$$\frac{25}{325} = e^{[-0.0056t]} \Rightarrow 0.0769 = e^{[-0.0056 \times t]}$$

$$\ln(0.0769) = -0.0056 \times t \Rightarrow -2.565 = -0.0056 \times t$$

$$t = \frac{2.565}{0.0056} = 458.03 \text{ sec.}$$

<u>systems with negligible surface resistance</u>



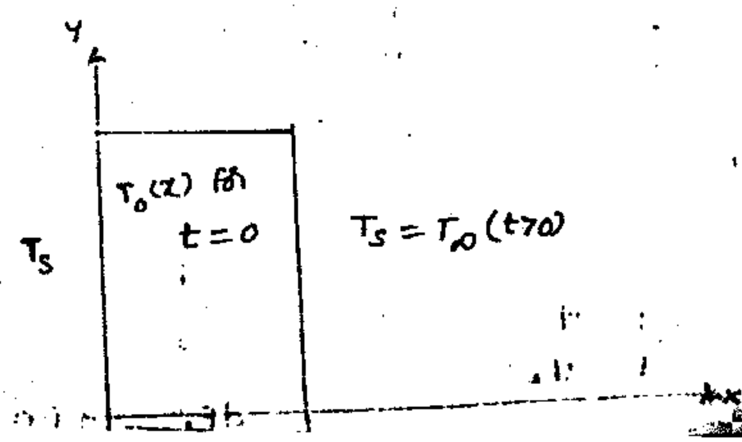
Internal resistance

⟶ conduction

surface resistance

⟶ convection

overall resistance = internal resistance (due to conduction) + surface resistance (due to convection)



$T_s = T_\infty (t > 0)$

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for 1-D general equation

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \quad 0 \le x \le L$$

Boundary conditions

at $t=0$, $T = T_0(x)$ for $0 \le x \le L$ (Initial condition)

$T = T_s$ at $x = 0$ for $t > 0$ } Boundary condition

$T = T_s$ at $x = L$ for $t > 0$

let us define $y = T - T_s$

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \Rightarrow \frac{\partial^2 y}{\partial x^2} = \frac{1}{\alpha}\frac{\partial y}{\partial t}$$

$y = y_0(x)$ at $t = 0$

$y = 0$ at $x = 0$ $t > 0$

$y = 0$ at $x = L$ $t > 0$

We consider y is a function of both variable x and time

$F, G \rightarrow$ Function

$$y = F(x) \cdot G(t)$$

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{\alpha}\frac{\partial y}{\partial t} \Rightarrow \frac{\partial^2[F(x) \cdot G(t)]}{\partial x^2} = \frac{1}{\alpha}\frac{\partial[F(x) \cdot G(t)]}{\partial t}$$

$$-\frac{1}{F}\frac{\partial^2 F}{\partial x^2} = -\frac{1}{\alpha}\frac{1}{G}\frac{\partial G}{\partial t}$$

let us consider a constant λ^2 equating to the both terms since F depends on x and G depends on t

$$-\frac{1}{F}\frac{d^2 F}{dx^2} = \lambda^2 \Rightarrow \frac{dF}{dx^2} = -F\lambda^2$$

$$\frac{dF^2}{dx^2} + F\lambda^2 = 0 \rightarrow \text{2nd order differential equation}$$

$$\frac{-dG}{dt} \times \frac{1}{G} \times \frac{1}{\alpha} = d^2 \Rightarrow \frac{dG}{dt} + d^2 \alpha G = 0$$

$\rightarrow$ First order differential equation

$F = A\cos dx + B\sin dx$, $G(t) = C\exp[-d^2\alpha t]$

$y = F(x) \cdot G(t) = (A\cos dx + B\sin dx) \cdot C\exp[-d^2\alpha t]$

$y = [CA\cos dx + CB\sin dx] \exp[-d^2\alpha t]$

$y = [C_1 \cos dx + C_2 \sin dx] \exp[-d^2\alpha t]$

$CA \rightarrow C_1$

$CB \rightarrow C_2$

By applying boundary conditions.

at $x = 0 \Rightarrow y = 0$, $t > 0$

$0 = [C_1 \cos d(0) + C_2 \sin d(0)] \exp(-d^2\alpha t)$

$C_1 \times 1 = 0 \Rightarrow C_1 = 0$

By substituting $C_1 = 0$

$y = [C_2 \sin dx] \exp(-d^2\alpha t)$

at $x = L \Rightarrow y = 0$

$0 = C_2(\sin dL) \exp(-d^2\alpha t)$

$C_2 = 0$ (or) $\sin dL = 0$

since the C_2 value cannot be zero for non-trivial solution

$\sin dL = 0 \Rightarrow \sin dL = \sin 0°$

$dL = n\pi$

$d = \frac{n\pi}{L}$ where $n = 1, 2, 3 + ----$

$y = \exp(-d^2\alpha t) \times C_2 \sin dL$

$$\psi = \sum_{n=1}^{\infty} C_n \left[\exp\left[\left(\frac{-n\pi}{L}\right)^2 \alpha t \right] \times \sin\left(\frac{n\pi}{L}\right) x \right]$$

at initial condition at $t=0$, $\psi = \psi_0(x)$

$$\psi_0 = \sum_{n=1}^{\infty} C_n \left[\exp\left[\left(\frac{-n\pi}{L}\right)^2 \times \alpha \times 0 \right] \sin\left(\frac{n\pi}{L}\right) \times x \right]$$

$$\psi_0 = \sum_{n=1}^{\infty} C_n \left[\sin\left(\frac{n\pi}{L}\right) x \right]$$

$$\left[\begin{matrix} n=1 & C_n = \frac{4\psi_0}{\pi} \\ n=2 & C_n = \frac{4\psi_0}{2\pi} \end{matrix} \right]$$

By fourier series of expansion

$$C_n = \frac{2}{L} \int_0^L \psi_0(x) \sin\left(\frac{n\pi}{L}\right) x \, dx = \frac{2\psi_0}{n\pi} [1 - \cos n\pi]$$

substituting the value of C_n in eq ①

$$\psi = \frac{2}{L} \int_0^L \psi_0(x) \sin\left(\frac{n\pi}{L}\right) x \, dx \sum_{n=1}^{\infty} \left[\exp\left(\left(\frac{-n\pi}{L}\right)^2 \alpha t \right] \times \sin\left(\frac{n\pi}{L}\right) x \right]$$

$$\psi = \frac{4\psi_0}{n\pi} \sum_{n=1}^{\infty} \left(\exp\left[\left(\frac{-n\pi}{L}\right)^2 \alpha t \right] \times \sin\left(\frac{n\pi}{L}\right) x \right]$$

$$\frac{T - T_s}{T_0 - T_s} = \frac{\psi}{\psi_0} = \frac{4}{n\pi} \sum_{n=1}^{\infty} \sin\left(\frac{n\pi}{L}\right) x \times \exp\left(\left(\frac{-n\pi}{L}\right)^2 \alpha t \right]$$

heat transfer rate $Q_{(x,t)} = -kA \frac{\partial T}{\partial x}$

$$Q = \frac{4kA}{n\pi} \psi_0 \times (T_s - T_0) \sum_{n=1}^{\infty} \frac{\left[\sin\left(\frac{n\pi}{L}\right) x \right] \exp\left(\left(\frac{-n\pi}{L}\right)^2 \alpha t \right]}{dx}$$

$$Q = \frac{4kA}{n\pi} \psi_0 (T_s - T_0) \sum_{n=1}^{\infty} \frac{n\pi}{L} \cos\left(\frac{n\pi}{L}\right) x \left[\exp\left(\frac{-n\pi}{L}\right)^2 \alpha t \right]$$

Total heat transfer $U = \int Q_{(x,t)} \, dt$

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$$U = \int_0^t \frac{4KA}{L}(T_s - T_0) \sum_{n=1}^{\infty} \cos\left(\frac{n\pi}{L}\right)x \times \exp\left[\left(\frac{-n\pi}{L}\right)^2 \alpha t\right]$$

$$U = \frac{4KA}{L}(T_s - T_0) \sum_{n=1}^{\infty} \left(\frac{-n\pi}{L}\right)^2 x \alpha t \;\exp\left[\left(\frac{-n\pi}{L}\right)^2 \alpha t\right] \cos\left(\frac{n\pi}{L}\right)x \Big]_0^t$$

$$U = \frac{4KA}{L}\alpha \times \left[\frac{-n^2\pi^2}{L^2}\right](T_s - T_0) \sum_{n=1}^{\infty}\left[\cos\left(\frac{n\pi}{L}\right)x \times 1 - \exp\left[\left(\frac{-n\pi}{L}\right)^2 \alpha t\right] \times \cos\left(\frac{n\pi}{L}\right)x\right]$$

$$U = \frac{4KA}{L} \cdot \frac{n^2\pi^2\alpha}{L^2}(T_0 - T_s) \sum_{n=1}^{\infty} \cos\left(\frac{n\pi}{L}\right)x\left[1 - \exp\left[\left(\frac{-n\pi}{L}\right)^2 \alpha t\right]\right]$$

① A large plate 50cm thick, initially has a uniform temperature of $40°c$. If the plate is suddenly raised to and maintained at a temperature of $440°c$ estimate (i) the temperature at a depth of 20cm from the surface after 30s (ii) The instantaneous heat flow across the above plane per m^2 after 30s (iii) the total heat flow across the above plane in 30s (iv) the temperature at the central plane after 30s Take $k = 6\,W/m\text{-}k$ and $\alpha = 2.5 \times 10^{-6}\,m^2/s$.

Sol: given that

Temperature distribution

$$\frac{T - T_s}{T_0 - T} = \frac{4}{\pi}\sum_{n=1}^{\infty} \frac{1}{n} \exp\left[-\left(\frac{n\pi}{L}\right)^2 \alpha t\right] \sin\left(\frac{n\pi x}{L}\right)$$

here $n = 1, 3, 5$

$$\left(\frac{n\pi}{L}\right)^2 \alpha t = \frac{n^2 \times \pi^2 \times 2.5 \times 10^{-6} \times 30}{0.5^2} = 0.003n^2$$

(i) at $x = 20cm = 0.2m$

$$\frac{x}{L} = \frac{0.2}{0.5} = 0.4$$

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(12)

$$\frac{T-T_s}{T_0-T_s} = \frac{4}{\pi}\frac{1}{n}\sum_{n=1}^{\infty} \exp[-0.003n^2]\sin\left(\frac{n\pi x}{L}\right)$$

here $n = 1, 3, 5$

taking only the first two terms

$$\frac{T-T_s}{T_0-T_s} = \frac{4}{\pi}\left[\frac{1}{1}\exp[-0.003(1)^2]\sin(1\times\pi\times 0.4) + \frac{1}{3}\{\exp[-0.003\times 3^2]\times\sin(3\times\pi\times 0.4)\}\right]$$

$$\frac{T-T_s}{T_0-T_s} = \frac{4}{\pi}\left[0.997\,(0.951) + \frac{1}{3}(0.973)\times(-0.588)\right]$$

$$\frac{T-T_s}{T_0-T_s} = \frac{4}{\pi}[0.948 - 0.191] = 0.757$$

$$T - T_s = 0.757\,(40-440) \Rightarrow T = T_s + 0.757[-400]$$

$$T = 440 + 0.757\,(-400)$$

$$= 440 - 303 = 137^\circ C$$

(ii) Instantaneous heat flow across the above plane @ 30°C

$$\frac{Q}{A} = \frac{4k}{L}(T_s - T_0)\sum_{n=1}^{\infty}\cos\left(\frac{n\pi x}{L}\right)\exp\left[-\left(\frac{n\pi}{L}\right)^2\alpha t\right], \; n=1,3,5$$

$$\frac{Q}{A} = \frac{4k}{L}(T_s - T_0)\sum_{n=1}^{\infty}\left[\exp\left[-\left(\frac{n\pi}{L}\right)^2\alpha t\right]\times\cos\left(\frac{n\pi x}{L}\right)\right]$$

$$\frac{Q}{A} = \frac{4\times 6\,[440-40]}{0.5}\left[\{\cos(1\times\pi\times 0.4)\,\exp[-0.003\times 1^2]\} + \cos(3\pi\times 0.4).\exp[-(9\times 0.003)]\right]$$

$$\frac{Q}{A} = \frac{4\times 6\times 400}{0.5}\left[0.999\,(0.997)\cdot 0.997\,(0.973)\right]$$

$$= 19200\,[0.996 - 0.970] = 1071.9 \ \ W/m^2$$

(iii) Total heat flow across the above plane in 30 sec

$$U(x,t) = \int_0^t Q(x,t)\,dt$$

$$= \frac{4}{\pi^2}\left(\frac{kAL}{\alpha}\right)(T_s - T_0)\sum_{n=1}^{\infty}\frac{1}{n^2}\left[1-\exp\left[-\left(\frac{n\pi}{L}\right)^2\alpha t\right]\right]\cos\left(\frac{n\pi x}{L}\right)$$

$n = 1, 3, 5$

$$U(0.2, 30) = \frac{4}{\pi^2}\left(\frac{6\times 1\times 0.5}{2.5\times 10^{-6}}\right)(440-40)\left[\frac{1}{1}\left(1-\exp[-1\times 0.003]\right)\cos(0.4\pi)\right.$$

$$\left.\frac{1}{9}\left[\left(1-\exp(-9\times 0.003)\right)\cos(3\pi\times 0.4)\right]\right]$$

$$U(0.2, 30) = 194.5\times 10^{6}\left[(1-0.997)(0.999) + \frac{1}{9}(1-0.973)(0.997)\right]$$

$$U(0.2, 30) = 194.5\times 10^{6}\left[2.994\times 10^{-3} - 2.953\times 10^{-3}\right]$$

$$= 7974.5\,W = 7.9745\,W$$

(iv) The temperature at the central plane after 30 sec

at $x = L/2$

$$\frac{T - T_s}{T_0 - T_s} = \frac{4}{\pi}\sum_{n=1,3}^{\infty}\frac{1}{n}\left[\exp\left(-\left(\frac{n\pi}{L}\right)^2\alpha t\right)\right]\sin\left(\frac{n\pi x}{L}\right)$$

at $x = L/2$

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$$\frac{T-T_s}{T_0-T_s} = \frac{4}{\pi}\sum_{n=1,3}^{\infty} \frac{1}{n}\left\{\left[\exp\left[-\left(\frac{n\pi}{L}\right)^2 \alpha t\right] \sin\left(\frac{n\pi}{2}\right)\right\}\right.$$

$$\frac{T-440}{40-440} = \frac{4}{\pi}\left[\frac{1}{1}\exp(-1\times0.003)\sin\left(\frac{\pi}{2}\right) - \frac{1}{3}\exp(-9\times0.003)\times \sin\left(\frac{3\pi}{2}\right)\right]$$

$$\frac{T-440}{40-440} = \frac{4}{\pi}\left[0.997 - 0.0267\right]$$

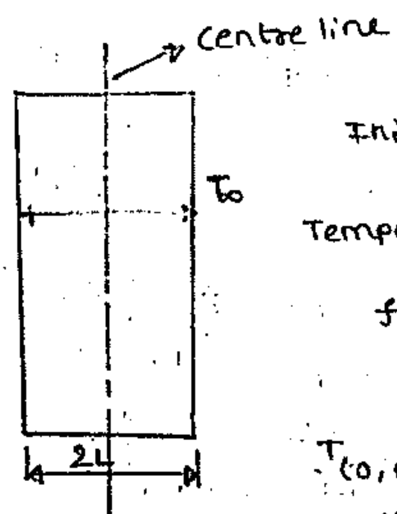
$$T-440 = -400\times\frac{4}{\pi}\left[0.9703\right]$$

$$T = 440 - \frac{400\times4}{\pi}(0.9703) = 54.2^\circ C$$

Heisler charts

* $B_i = \frac{hL_c}{k} < 0.1$ = Lumped heat capacitance is possible

$B_i > 0.1$ = Lumped heat analysis is not possible



Initial Temperature $= T_0$

Temperature is depend upon (or)

function $(x, t) \rightarrow B_i, F_0, x/L$

$T_{(0,t)} \rightarrow$ centre line temperature at time (t)

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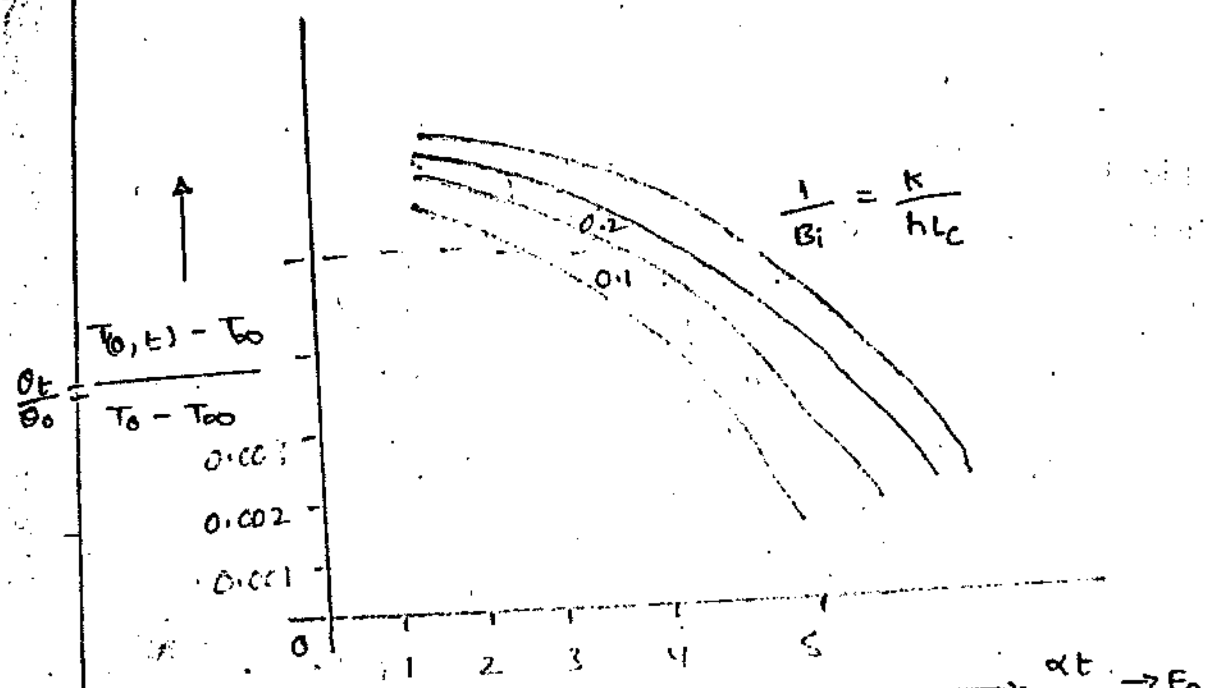
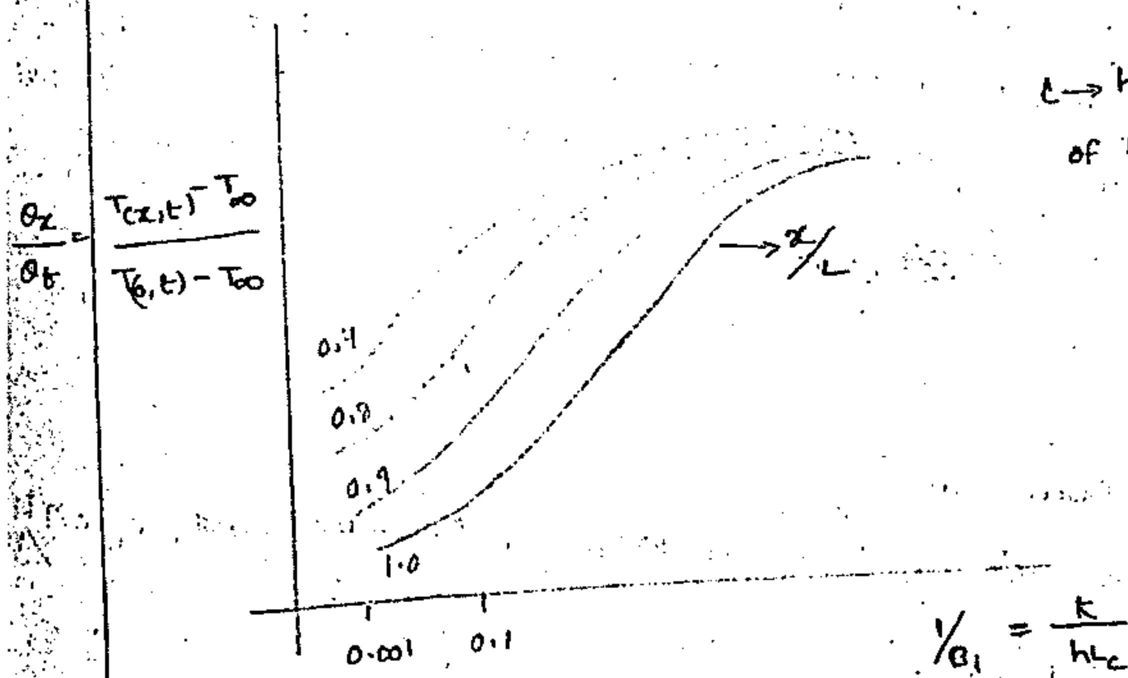


Fig(a)

This graph says that centre line Temperature distribution (vs) Fourier number i.e Fig(a)

$l \rightarrow$ half thickness of the wall



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$$\frac{1}{B_i} = \frac{1}{0.27} \simeq \frac{1}{0.3} = 3.583$$ $t = 1\,min = 60\,sec$

$$\frac{\alpha t}{L^2} = \frac{8.4\times10^{-5}\times 60}{(0.05^2)} = 2.04$$

Temperature at centre line $\frac{T_0 - T_\infty}{T_i - T_\infty} = 0.6 \rightarrow @\left[\frac{\alpha t}{L^2}, \frac{1}{B_i}\right]$

$T_i = 500°c$, $T_\infty = 100°c$

$$(T_0 - 100) = 0.6\,[500 - 100]$$

$$T_0 = 240°c$$

Temperature at surface $\frac{x}{L} = 1$ and for $\frac{1}{B_i} = \frac{k}{hL}$
$= 3.583$

$$\frac{T_{(x,t)} - T_\infty}{T_{(0,t)} - T_\infty} = 0.880 \Rightarrow T_{(x,t)} = 0.88\,[372 - 100] + 100$$
$$= 339.36°c$$

or

To calculate the energy loss

$$\frac{h^2 \alpha t}{k^2} = \frac{1200^2 \times (8.4\times10^{-5})\times 60}{215^2} = 0.157$$

$$B_i = \frac{hL}{k} = \frac{1200\times0.05}{215} = 0.28$$

From data book

$$\frac{U}{U_0} = 0.32 \Rightarrow \frac{U_0}{A} = \frac{\rho c V (T_0 - T_\infty)}{A} = \rho c\, 2L\,(T_0 - T_\infty)$$

$$= 2700 \times 900 \times 0.1 \times 400 = 97.2\times10^6\, J/m^2$$

heat removed per unit surface area is

$$\frac{U}{A} = 0.32 \times 97.2\times10^6 = 31.1\times10^6\, J/m^2$$

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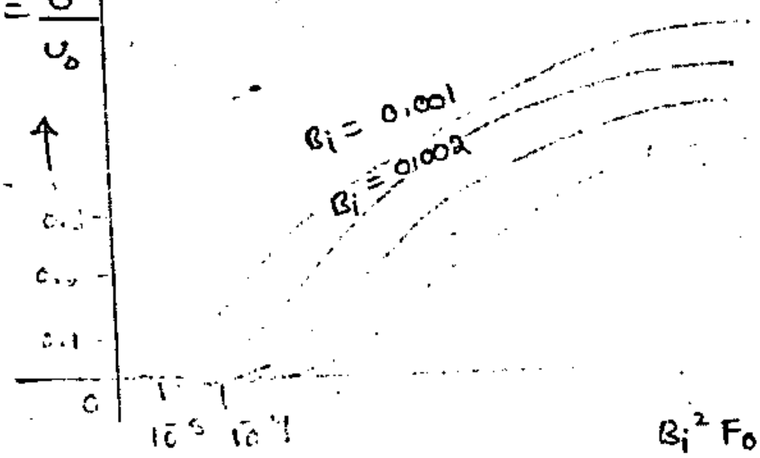
$$\frac{\text{Actual heat loss/gain}}{\rho c V (T_0 - T_\infty)} = \frac{U}{U_0}$$



$$B_i^2 F_0 = \frac{h^2 \alpha t}{k^2}$$

① A slab of Aluminium 10cm thick is originally in a temperature of 500°C. It is suddenly immersed in a liquid at 100°C resulting in a heat transfer coefficient of 1200 W/m²K. Determine the temperature at the centreline and the surface 1 minute after the immersion. Also calculate the total thermal energy removed per unit area of the slab during this period. The properties of Aluminium for given condition $\alpha = 8.4 \times 10^{-5}$ m²/s, $k = 215$ W/mK, $\rho = 2700$ kg/m³, $c = 0.9$ kJ/kg-K

Sol:- given that

$\alpha = 8.4 \times 10^{-5}$ m²/s, $k = 215$ W/mK, $\rho = 2700$ kg/m³, $c = 0.9$ kJ/kgK

W.K.T $B_i = \frac{hL_c}{k}$

$2L = 10$ cm $= 0.1$ m

$L = 0.05$ m

$B_i = \frac{1200 \times 0.05}{215}$

$= 0.27 70.1$

$= 0.3$

(21)

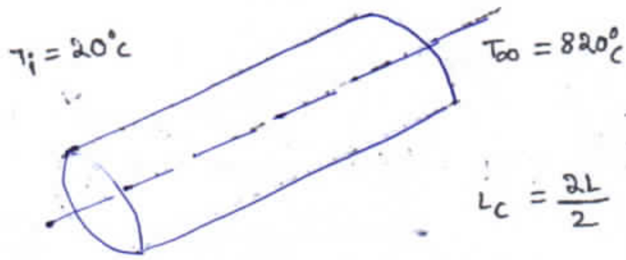
② A long steel cylinder 12.0cm in diameter and initially at 20°c is placed into a furnace at 820°c with the local heat transfer coefficient $h = 140 W/m^2K$. Calculate the time required for the axis temperature to reach 800°c. Also calculate the corresponding temperature at a radius of 5.4 cm at that time. The physical properties of steel are $\alpha = 6.11\times10^{-6} m^2/s$, $k = 21 W/mK$

Sol:- given that

$\alpha = 6.11\times10^{-6} m^2/s \quad K = 21 W/mK, \; T_0 = 20°c, \; T_\infty = 820°c$

$d = 12, \; r_0 = 6cm = 0.06m$

$T_0 = T_i = 20°c, \; T_{(0,t)} = 820°c$



$L_c = \frac{2L}{2}$

$r_c = \frac{r_0}{2}$

$$B_i = \frac{h r_0}{2k} = \frac{140\times0.06}{21\times2}$$

$$= 0.2 > 0.1$$

$$\frac{T_{(0,t)} - T_\infty}{T_0 - T_\infty} = \frac{800-820}{20-820} = 0.025$$

From data book $\left(\frac{1}{2B_i}\right) = \frac{1}{2\times0.2} = \frac{1}{0.4} = 2.5$

$$F_0 = \frac{\alpha t}{r_0^2} \Rightarrow \frac{6.11\times10^{-6}\times t}{(0.06)^2} = 7$$

$$t = \frac{0.06^2\times7}{6.11\times10^{-6}} = 4124.38 sec = 68 min \; 33 sec$$

at 5.9 cm radius and $t = 4124.38 sec$, $T = ?$

$t=0, \; T = T_0$

$$\frac{T - T_\infty}{T_0 - T_\infty} = \frac{T-820}{20-820}$$

$T_{(r,t)}$, $t=0$

$x=0, \; t>0 \quad T = T_{(0,t)}$

$x=r, \; t>0 \quad T = [T_{(r,t)}]$

$$\frac{r}{r_0} = \frac{5.4}{6} = 0.9$$

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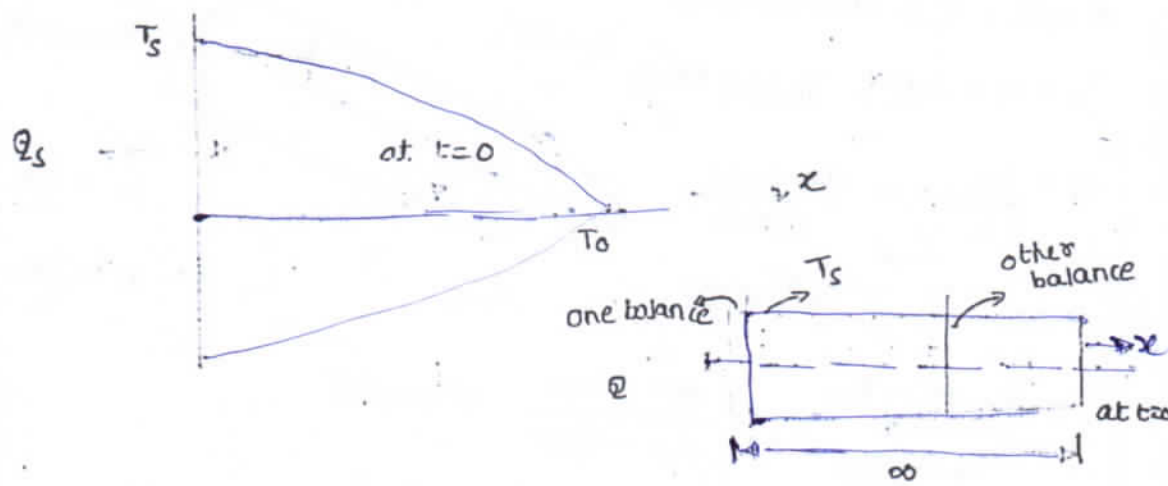
$$\frac{T-820}{20-820} = 0.84$$ (from databook @ $B_i = 0.4$ and $\frac{r}{r_0} = 0.9$

$$T - 820 = 0.84(20-820)$$

$$T = 820 + 0.84(-800) = 148^{\circ}C$$ → at same rod at

$T_i = 20^{\circ}C$

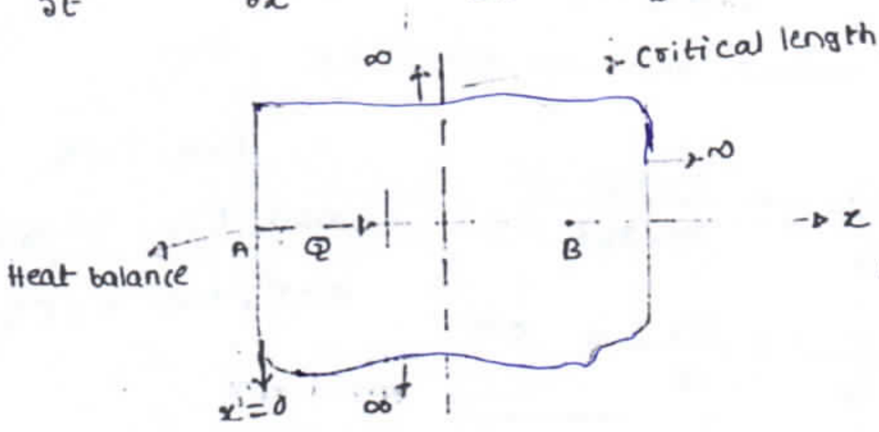
Heat Flow through semi infinite body



$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \Rightarrow \frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2}$$

$$\frac{\partial \theta}{\partial t} = \alpha \frac{\partial^2 \theta}{\partial x^2} \Rightarrow \frac{\Delta T}{\Delta t} = \alpha \frac{\Delta T}{x^2}$$



$$g = \frac{1}{\sqrt{-2c_1\alpha t}} \qquad g(t) \sim \frac{1}{\sqrt{\alpha t}} = \frac{1}{x_{ref}}$$

$c_1 = -2$ or $-1/2$

take $c_1 = -2$

$$g = \frac{1}{2\sqrt{\alpha t}}$$

$$\frac{\frac{d^2T}{d\eta^2}}{\eta \frac{dT}{d\eta}} = -2 \Rightarrow \frac{d^2T}{d\eta^2} = -2\eta \frac{dT}{d\eta}$$

$$\frac{d^2T}{d\eta^2} + 2\eta \frac{dT}{d\eta} = 0$$

let $\frac{dT}{d\eta} = F \Rightarrow \frac{dF}{d\eta} + 2\eta F = 0$

$$\frac{dF}{F} + 2\eta \, d\eta = 0$$

on integrating $\Rightarrow \ln F + 2 \times \frac{\eta^2}{2} = \ln c_3$

$$\ln F - \ln c_3 = -\eta^2$$

$$\ln\left[\frac{F}{c_3}\right] = -\eta^2$$

$$\boxed{\frac{dT}{d\eta} = F}$$

$$\frac{F}{c_3} = e^{-\eta^2}$$

$$F = c_3 \times e^{-\eta^2} \Rightarrow \frac{dT}{d\eta} = c_3 e^{-\eta^2}$$

$$\frac{dT}{d\eta} = c_3 e^{-\eta^2} \Rightarrow dT = c_3 e^{-\eta^2} d\eta$$

on integrating $T = c_3 \int_0^{\eta} e^{-\eta^2} d\eta + c_4$

Applying the boundary condition at $x = 0 \rightarrow T = T_0 \rightarrow \eta = 0$

$\boxed{C_4 = T_0}$ $\qquad \eta = \dfrac{x}{2\sqrt{\alpha t}}$

Applying the Initial condition

at $t = 0$, $\eta = \infty$, $T = T_i$

$$T_i = c_3 \int_0^{\eta} e^{-\eta^2} d\eta + T_0$$

$$\int_0^{\infty} e^{-\eta^2} d\eta \quad \Rightarrow \text{ let } \eta^2 = y$$

$$\eta = y^{1/2}$$

$$d\eta = \frac{1}{2} y^{\left(\frac{1}{2}-1\right)}$$

$$\Rightarrow \frac{1}{2}\int_0^{\infty} e^{-y} \times y^{\left(\frac{1}{2}-1\right)} \longrightarrow \text{This is called gama of half}$$

$$\Gamma(1/2) = \sqrt{\pi}$$

$$\frac{1}{2}\sqrt{\pi}$$

$$T_i = c_3 \times \frac{\sqrt{\pi}}{2} + T_0 \Rightarrow \frac{(T_i - T_0)2}{\sqrt{\pi}} = c_3$$

$$T = c_3 \int_0^{\eta} e^{-\eta^2} d\eta + C_4 \Rightarrow T = (T_i - T_0)\frac{2}{\sqrt{\pi}} \int_0^{\eta} e^{-\eta^2} d\eta + T_0$$

$$(T_i - T_0) = (T_i - T_0)\frac{2}{\sqrt{\pi}} \int_0^{\eta} e^{-\eta^2} d\eta$$

$$\frac{(T - T_0)}{(T_i - T_0)} = \frac{2}{\sqrt{\pi}} \int_0^{\eta} e^{-\eta^2} d\eta \longrightarrow \text{error function } (y)$$

where $\eta = \dfrac{x}{2\sqrt{\alpha t}}$

$$\frac{(T - T_0)}{(T_i - T_0)} = \operatorname{erf}(\eta)$$

W.r.t heat (Temperature) is a function of space and time

1-D transient heat conduction equation $\frac{1}{\alpha}\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2}$

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2}$$

initial condition at $t=0 \rightarrow T = T_i$ [for all x]

Boundary condition at $x=0 \Rightarrow T = T_0$ $(t>0)$

$$\frac{\Delta T}{\Delta t} = \alpha \frac{\Delta T}{x^2_{reference}}$$

$x_{reference} \longrightarrow$ The position at which the heat that can sense

$$\frac{\Delta T}{t_{ref}} = \alpha \frac{\Delta T}{x^2_{ref}}$$

$$\boxed{t_{ref} = t}$$

$$x^2_{ref} = \alpha t \Rightarrow x_{ref} = \sqrt{\alpha t}$$

Non-dimension parameter

$$\frac{x}{x_{ref}} = \frac{x}{\sqrt{\alpha t}} = x \times \frac{1}{\sqrt{\alpha t}} = x \cdot g(t)$$

→ assumption

consider as similarity parameter

$$\eta = x \cdot g(t)$$

$$\frac{\partial T}{\partial t} = \frac{\partial T}{\partial \eta} \times \frac{\partial \eta}{\partial t} = \frac{dT}{d\eta} \times x \times \left[\frac{d\,g(t)}{dt}\right]$$

$$= x g' \frac{dT}{d\eta}$$

$$\frac{\partial T}{\partial x} = \frac{\partial T}{\partial \eta} \times \frac{\partial \eta}{\partial x} = \frac{dT}{d\eta} \times g(t)$$

$$\frac{\partial^2 T}{\partial x^2} = \frac{\partial}{\partial x}\left[\frac{dT}{d\eta} \times g\right] = \frac{d}{d\eta}\left[\frac{\partial T}{\partial x}\right] \times g$$

$$= \frac{d}{d\eta}\left[\frac{dT}{d\eta} \times \frac{\partial \eta}{\partial x}\right] \times g$$

$$= \frac{d}{d\eta}\left[\frac{dT}{d\eta} \times g\right] \times g$$

$$\frac{\partial^2 T}{\partial x^2} = g^2 \frac{d^2 T}{d\eta^2}$$

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2} \Rightarrow x \cdot g' \frac{dT}{d\eta} = \alpha \times g^2 \frac{d^2 T}{d\eta^2}$$

$$\eta = x \cdot g \Rightarrow x = \frac{\eta}{g} \Rightarrow \frac{\eta}{g} \times g' \frac{dT}{d\eta} = \alpha g^2 \frac{d^2 T}{d\eta^2}$$

$$\frac{d^2 T}{d\eta^2} \times g^3 = \eta g' \frac{dT}{d\eta}$$

$$\frac{\frac{d^2 T}{d\eta^2}}{\eta \frac{dT}{d\eta}} = \frac{g'}{\alpha g^3} = C$$

function of "η" function of time

$$\frac{g'}{\alpha g^3} = C_1 \Rightarrow \frac{\frac{dg}{dt}}{\alpha g^3} = C_1$$

$$g^{-3} dg = \alpha C_1 \times dt$$

on integration $\Rightarrow \frac{g^{-2}}{-2} = \alpha C_1 t + C_2$

$$\frac{x}{x_{ref}} = x \cdot g(t) \Rightarrow \frac{1}{x_{ref}} \sim g(t)$$

at $t \to 0 \Rightarrow x_{ref} \to \infty$

$$\frac{-1}{2(\infty)} = \alpha C_1 \times 0 + C_2 \Rightarrow C_2 = 0$$

$$\frac{g^{-2}}{-2} = \alpha C_1 t \Rightarrow \frac{1}{g^2} = -2C_1 \alpha t$$

$$g^2 = \frac{1}{-2C_1 \alpha t} \Rightarrow g = \frac{1}{\sqrt{-2C_1 \alpha t}}$$

① A semi-infinite slab of copper is exposed to a constant heat flux at the surface of 0.3 mW/m^2. Neglecting convection at the surface calculate surface temperature after 10 minutes if the initial temperature of the slab is 30°c. What is the temperature at a distance of 20cm from the surface after 10min for copper $k = 386$ W/m-k, $\alpha = 0.404$ m^2/hr

Sol:- given that

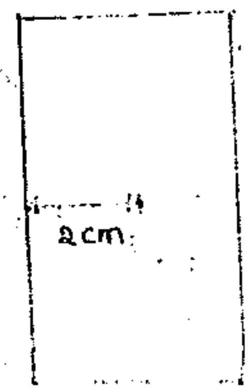
for copper $k = 386$ W/m-k, $\alpha = 0.404$ m^2/hr, $x = 20cm = 0.2m$

$T_i = 30°c$, $q_0 = 0.3$ mW/m^2 $= 0.3 \times 10^6$ W/m^2

$q_0 \rightarrow$ heat flux at surface

Surface at $t = 0$

$T_0 = ?$



$$q_0 = \frac{k(T_0 - T_i)}{\sqrt{\pi \alpha t}}$$

where $T_0 \rightarrow$ surface temperature

$T_i \rightarrow$ Initial Temperature

$$q_0 = \frac{k(T_0 - T_i)}{\sqrt{\pi \alpha t}} = \frac{386(T_0 - 30)}{\sqrt{\pi \times 0.404 \times (10/60)}}$$

$$0.3 \times 10^6 = \frac{386(T_0 - 30)}{\sqrt{\pi \times 0.404 \times (10/60)}}$$

$$0.3 \times 10^6 \times \sqrt{\pi \times 0.404 \times \left(\frac{10}{60}\right)} = 386(T_0 - 30)$$

$T_0 = 387.45°c$ at $t = 10min$

At $t = 10$ min at a distance of 20 cm.

$$\frac{T_x - T_0}{T_i - T_0} = erf(z) \qquad \text{where } z = \frac{x}{2\sqrt{\alpha t}} = \frac{0.2}{2\sqrt{0.404 \times \left(\frac{10}{60}\right)}}$$

$z = 0.385$

$erf(z) = erf(0.385) = 0.41879$ (from data book)

$$\frac{T_x - 387.45}{30 - 387.45} = 0.41879 \Rightarrow T_x = 237.77°c$$

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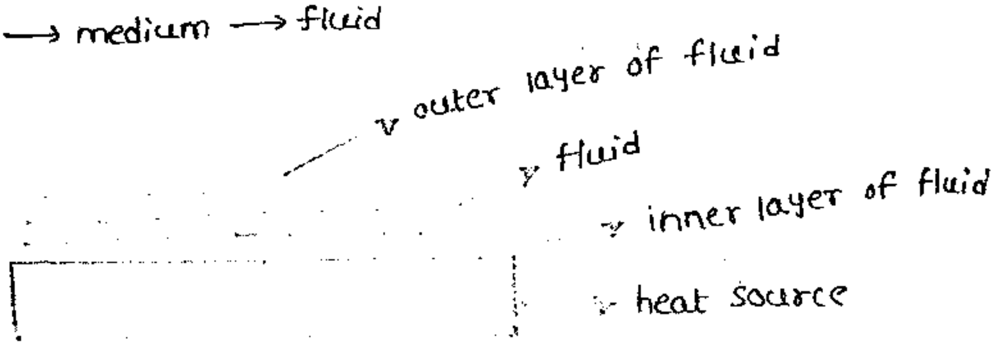
CONVECTIVE HEAT TRANSFER UNIT-3

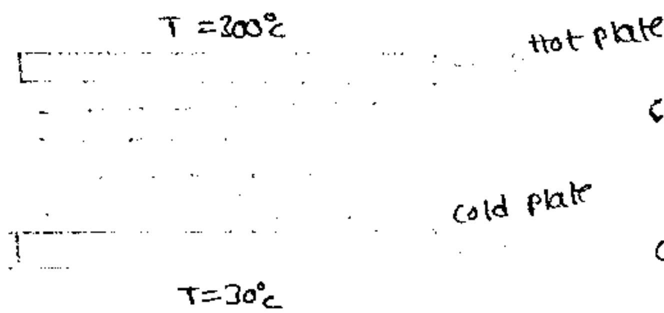
Convection :-

It is the mode of heat transfer between the surface and fluid moving over it

Conduction ⟶ medium ⟶ solid

convection ⟶ medium ⟶ fluid





conduction ⟶ fluid is rest (molecular interaction)

Convection ⟶ fluid is in motion (molecular momentum)

classification of convective heat transfer

It is two types

① Natural or Free convection ⟶ due to the buoyancing effect

② Forced convection ⟶ super imposed by some velocity field, fan, blower pump

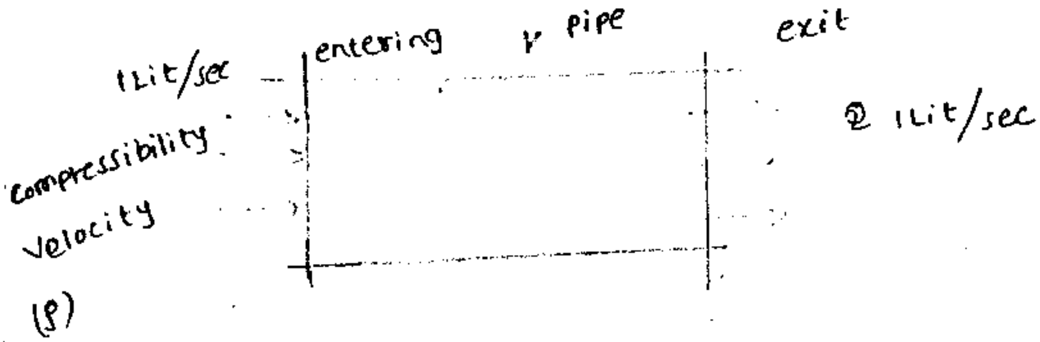
Natural & free convection } fluid is in motion
Forced convection

* Fluid is in motion - the temperature field is determined by considering the dynamics of field motion

* Flow of viscous fluids over the plates (external flows) and in the (Internal flows) tubes

continuity equation

The continuity equation is essential to determine the conservation of mass

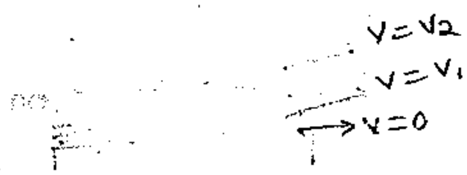


heat transfer through convection

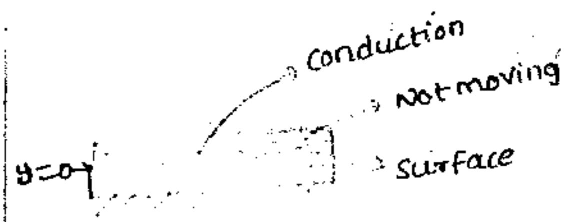
no slip condition

The fluid at the surface is sticked when the fluid is in motion, the fluid stuck at the surface due to friction this is called no slip

no – temperature inmpression condition



$V_2 > V_1 > V$



$$Q_{conv} = Q_{cond} = -kA \left.\frac{\partial T}{\partial y}\right|_{y=0}$$

T is the temperature, $\frac{\partial T}{\partial y} \rightarrow$ Temperature gradient

$$Q_{cond} = -kA\left[\frac{\partial T}{\partial y}\right]_{y=0}$$

where $h \rightarrow$ Local heat transfer coefficient

$$Q_{conv} = hA\,(T_s - T_f) = hA(T_s - T_\infty)$$

$$hA(T_s - T_\infty) = -kA\left(\frac{\partial T}{\partial y}\right)\bigg|_{y=0}$$

$$h = \frac{-kA\left(\frac{\partial T}{\partial y}\right)_{y=0}}{A(T_s - T_\infty)} \Rightarrow h = \frac{-k\cdot\left(\frac{\partial T}{\partial y}\right)_{y=0}}{(T_s - T_\infty)}$$

mean & average heat transfer coefficient

$$\bar{h}_L = \frac{1}{L}\int_0^L h\,dx$$

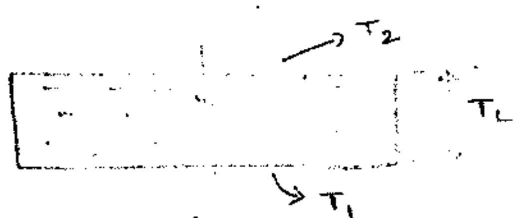
Nusselt number

The nusselt number is the ratio of convective to conductive heat transfer across boundary

$$Nu = \frac{hL_c}{k}$$ where $k \rightarrow$ thermal conductivity of fluid

$L_c \rightarrow$ characteristic length

$h \rightarrow$ convective heat transfer coefficient



$$q_{cond} = k\frac{\Delta T}{L}$$

$$q_{conv} = h\Delta T$$

$$\Delta T = (T_2 - T_1)$$

$$\frac{q_{conv}}{q_{cond}} = \frac{h\Delta T}{k\frac{\Delta T}{L}} = \frac{hL}{k} = Nu$$

* heat transfer by convection to the heat transfer by conduction

(No-slip condition) $Nu = 1$ $\quad q_{conv} = q_{cond}$

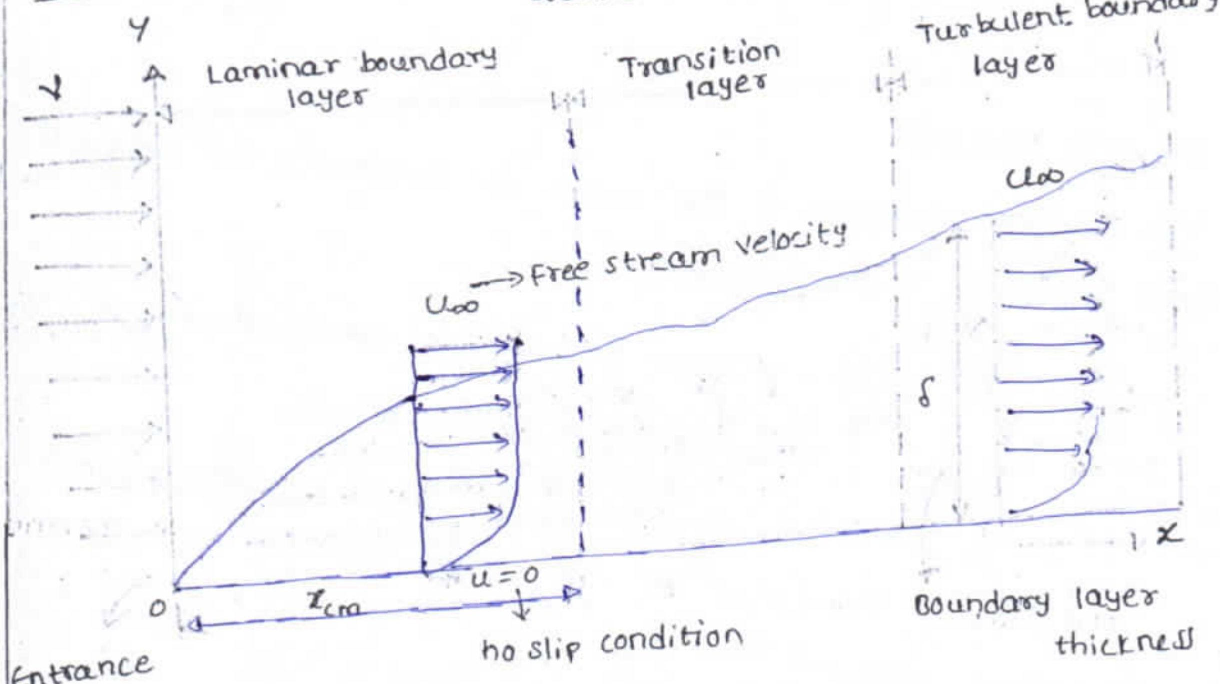
$Nu = 1 \rightarrow$ heat transfer $\rightarrow$ pure conduction

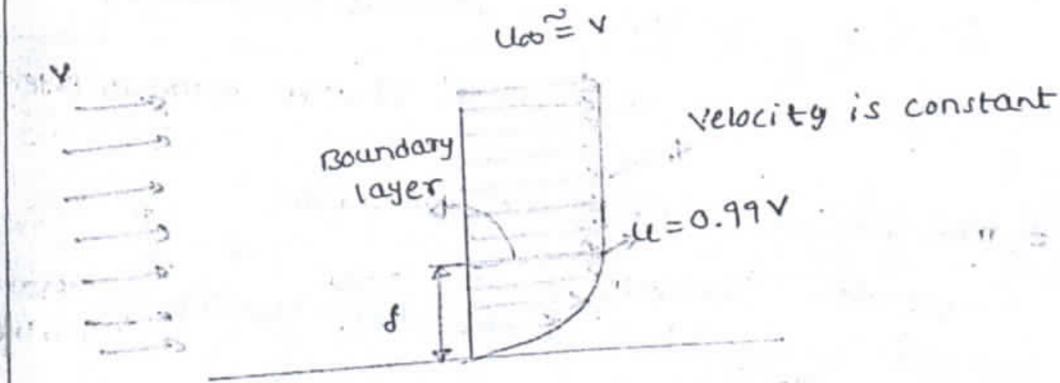
$Nu > 1 \rightarrow$ larger the effective convection heat transfer

classification of fluid Flows

1. viscous and inviscid flow
2. Internal and external flow
3. compressible and incompressible flow
4. laminar and turbulent Flow
5. natural and forced flow
7. steady and unsteady Flow
8. one, two and three-dimensional Flow

Velocity boundary layer → Fluid layer bounded





boundary layer thickness

Shear stress in Fluid Flow

$$\tau \propto \left.\frac{\partial u}{\partial y}\right|_{y=0}$$

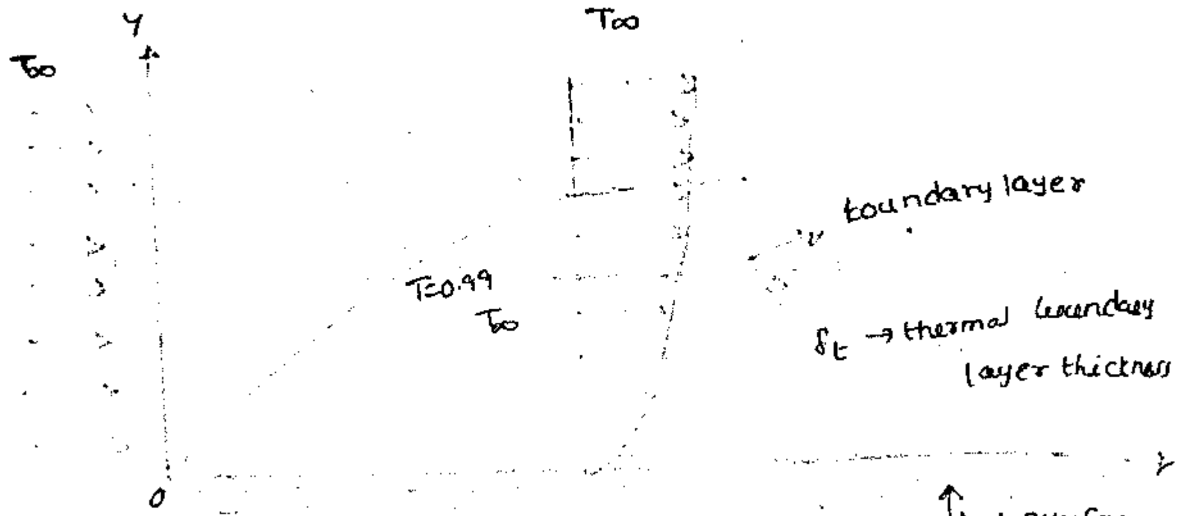
$$\tau = \mu \left.\frac{\partial u}{\partial y}\right|_{y=0}$$

where μ = dynamic viscosity
= $\rho \times$ kinematic viscosity

$$\tau = C_f \times \frac{\rho v^2}{2} \longrightarrow \text{kinetic energy}$$

$C_f \rightarrow$ friction coefficient

Thermal boundary layer



$\delta_t \rightarrow$ thermal boundary layer thickness

↑ hot surface

$$(T - T_s) \simeq 0.99\,(T_\infty - T_s)$$

Prandtl number

The relative thickness of the velocity and the thermal boundary layer

$$P_r = \frac{\delta}{\delta_t} = \frac{\text{Thickness of velocity boundary layer}}{\text{Thickness of thermal boundary layer}}$$

$$P_r = \frac{\text{molecular diffusivity of the momentum}}{\text{molecular diffusivity of the heat}} = \frac{\nu}{\alpha}$$

$\alpha \rightarrow$ thermal diffusivity

$$P_r = \frac{\nu}{\alpha} = \nu\left(\frac{1}{\alpha}\right) \qquad \frac{1}{\alpha} = \frac{\rho C_p}{k}$$

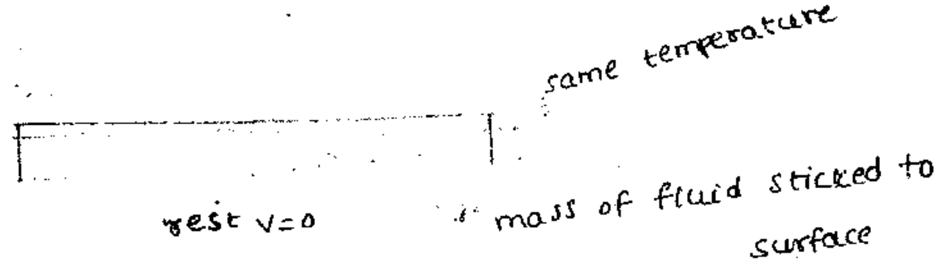
$$P_r = \nu \times \frac{1}{\alpha} = \nu \times \frac{\rho C_p}{k} = \frac{\mu C_p}{k}$$

$\mu \rightarrow$ dynamic viscosity

$$P_r = \frac{\mu C_p}{K}$$

continuity equation

The continuity equation is essentially the equation for the conservation of mass (mass neither be created nor destroyed)

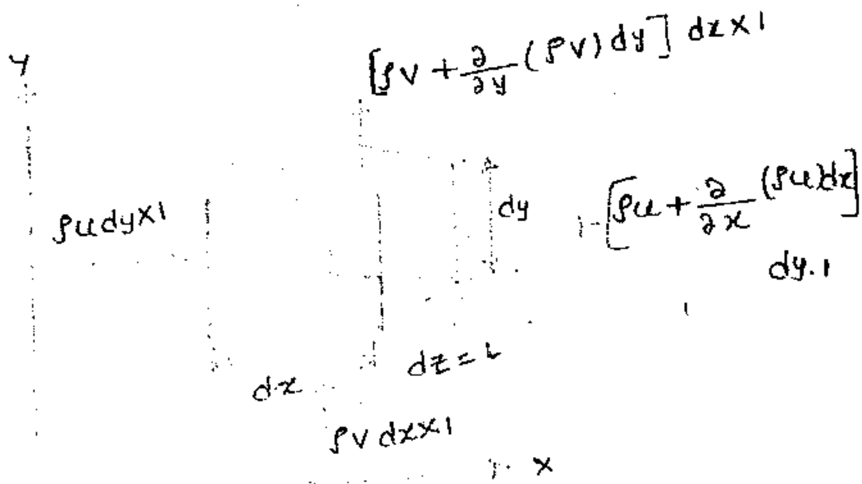


w.r.t

$\rho = m/v$

$\frac{m}{s} = \frac{\rho \times v}{s} = \frac{\rho \times A \times l}{s}$

$= \rho \times A \times v$



$u \rightarrow$ velocity component in x-direction $v \rightarrow$ velocity component in y-direction

Assume: ① Flow is steady ② Flow is incompressible $\Rightarrow \rho =$ constant

The mass flow into the infinitesimal volume $dx \cdot dy \cdot 1$ must be equal to the mean flow out of the volume

$$(\rho u dy) + [\rho v dx] = [\rho u + \frac{\partial}{\partial x}(\rho u) dx] dy + [\rho v + \frac{\partial}{\partial y}(\rho v) dy] dx$$

Divide with $dxdy$ both sides

$$\frac{\rho u dy}{dxdy} + \frac{\rho v dx}{dxdy} = \rho u \frac{dy}{dxdy} + \frac{\partial}{\partial x} \rho u \frac{dxdy}{dxdy} + \rho v \frac{dx}{dxdy}$$

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$$+\frac{\partial}{\partial y}(\rho v)\ \frac{dy\,dx}{dx\,dy}$$

$$\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial y}(\rho v)=0$$

But we consider $\rho=$ constant

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$ → continuity equation f8 2-D flow

For three-Dimensional Flow

$$\text{div}\ \rho V=0\ ;\qquad V=i_1 v_1+i_2 v_2+i_3 v_3$$

For axis symmetric cylindrical / polar coordinates

$$\underbrace{\frac{\partial v_r}{\partial r}+\frac{v_r}{r}+\frac{\partial v_z}{\partial z}}=0$$ → 3-D continuity equation f8 axis-symmetrical model

r, z → coordinates (polar)

<u>Dimensional Analysis</u>

Fundamental Quantities (units)

derived (or) physical quantities (units)

mass, length, time, Temp

	unit	Dimensional number
mass	kg	M
length	m	L
Time	sec	T
Temp	°K/°c	θ

	unit	Dimensional unit
Force	N	

$F = ma = kg\ m/s^2 = \frac{M\times L}{T^2} = MLT^{-2}$

pressure (p) $= F/A = \frac{MLT^{-2}}{L\times L} = ML^{-1}T^{-2}$

Kinematic viscosity ν $m^2/s = L^2T^{-1}$

velocity ⇒ v ⇒ $m/s = LT^{-2}$

Dimensionless number

① Biot number → $\frac{\text{Internal conduction resistance}}{\text{surface convection resistance}} = \frac{hL}{k}$

$$h \rightarrow \frac{W}{m^2K} \rightarrow \frac{Nm/s}{m^2K} = \frac{N}{msK} = \frac{kgm/s^2}{msK} = \frac{kg}{s^3K}$$

$$= MT^{-3}\theta^{-1}$$

$$L \rightarrow m \rightarrow L, \quad k \rightarrow \frac{W}{mK} = \frac{Nm/s}{mK} = \frac{N/s}{K} = \frac{kg\,m/s^2}{sK}$$

$$= \frac{kgm}{s^3K} = MLT^{-3}\theta^{-1}$$

$$\frac{hL}{k} = \frac{MT^{-3}\theta^{-1} \times L}{MLT^{-3}\theta^{-1}} = 1$$

② Euler number → $\frac{\text{pressure force}}{\text{Inertia force}} = \frac{\Delta p}{\rho u^2}$

③ Fourier number → $\frac{\text{characteristic body dimension}}{\text{Temperature wave penetration depth in time "t"}} = \frac{\alpha T}{L^2}$

④ Froude number → $\frac{\text{Inertia force}}{\text{gravity force}} = \frac{u^2}{gL}$

⑤ Graetz number → $\frac{\text{heat transfer by conduction}}{\text{heat transfer by convection}} = Re \times Pr \times \frac{d}{x}$

⑥ Peclet number → $\frac{\text{heat transfer by convection}}{\text{heat transfer by conduction}} = RePr = \frac{uLc_p}{k}$

⑦ Nusselt number → Ratio of temperature gradient by conduction and convection at surface

$$= Nu = \frac{hL}{k}, \quad \frac{hd}{k} \rightarrow \text{tubes \& pipes}$$

↳ surface

⑧ Grashof number

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Grashof number = $\left[\dfrac{\text{Buoyant force}}{\text{viscous force}}\right]\left[\dfrac{\text{Inertia force}}{\text{viscous force}}\right]$

$$= Gr = \frac{g\beta\Delta T L^3 \rho^2}{\mu^2}$$

(9) knudson number → $\dfrac{\text{mean free path}}{\text{characteristic body dimension}} = K_n = \lambda/L$

(10) Lewis number = $\dfrac{\text{heat diffusivity}}{\text{mass diffusivity}} = L_e = \alpha/D$, $\dfrac{S_c}{P_r}$

(11) prandtl number = $\dfrac{\text{molecular diffusivity of momentum}}{\text{molecular diffusivity of heat}}$

$$= P_r = \frac{C_p \mu}{k}$$

(12) Reyleigh number = product of Grashof number and prandtl number

$$Gr\,Pr = \frac{g\beta\Delta T L^3 \rho^2 C_p}{\mu k}$$

(13) Reynolds number = $\dfrac{\text{Inertia force}}{\text{viscous force}} = \dfrac{uL}{\nu}$, $\dfrac{uL\rho}{\mu}$

(14) Schmidt number = $\dfrac{\text{molecular diffusivity of momentum}}{\text{molecular diffusivity of mass}}$

$$S_c = \frac{\mu}{\rho D} = \frac{\nu}{D}$$ → kinematic viscosity

(15) Sherwood number = Ratio of concentration gradient at the boundary by diffusion in by convection

$S_h \rightarrow h_m L/D$, $\dfrac{h_m d}{D}$

(16) stanton number = $\frac{\text{wall heat transfer rate}}{\text{heat transfer by convection}}$

$$S_t = \frac{Nu}{Re\,Pr} = \frac{h}{c_p \rho u}$$

Dimensional analysis

Rayleigh method

variables from (3 to 4)

Buckingham π theorem

variables from > 4 variables

*Dimensional Analysis of stress

$$\sigma = \frac{\text{load}}{\text{Area}} = \frac{\text{Force}}{\text{Area}} = \frac{N}{L^2} = \frac{kgm/s^2}{m^2} = ML^{-1}T^{-2}$$

σ = function (force , Area)

$\sigma = f[F, A]$

$N \rightarrow \frac{kgm}{s^2} = MLT^{-2}$

LHS RHS

$$ML^{-1}T^{-2} = f[F^a \times A^b]$$

$$ML^{-1}T^{-2} = f\left[(MLT^{-2})^a \cdot (L^2)^b\right]$$

$$ML^{-1}T^{-2} = [M^a L^a T^{-2a} \cdot L^{2b}] = [M^a L^{a+2b} T^{-2a}]$$

$M^1 = M^a \Rightarrow a = 1$, $L^{-1} = L^{a+2b} \Rightarrow a + 2b = -1$

$1 + 2b = -1$

$2b = -2$

$b = -1$

$T^{-2} = T^{-2a} \Rightarrow -2 = -2a$

$\boxed{a = 1}$

$$ML^{-1}T^{-2} = f[F^a \cdot A^b] = [F^1 A^{-1}] =$$

① Find the expression for the dragforce on smooth sphere of diameter D moving with uniform velocity "u" in a fluid having density "ρ" and dynamic viscosity "μ"

Sol:- let the drag force F

$$F = f(D, u, \rho, \mu)$$

$$F = C(D^a, u^b, \rho^c, \mu^d)$$

$$MLT^{-2} = C(D^a, u^b, \rho^c, \mu^d) \longrightarrow ①$$

$D \rightarrow$ Diameter $\rightarrow m \rightarrow L$ $\quad u \rightarrow$ velocity $\rightarrow m/s \rightarrow LT^{-1}$

$\rho \rightarrow$ density $\rightarrow kg/m^3 \rightarrow ML^{-3}$ $\quad \mu \rightarrow$ dynamic viscosity $\rightarrow \frac{N\text{-}s}{m} = ML^{-1}T^{-1}$

$$\mu = \rho \times u$$

$$= \frac{kg}{m^3} \times \frac{m^2}{s} \times \frac{s}{s} = \left[\frac{kg\,m}{s^2}\right] \times \frac{s}{m^2} = \frac{N\text{-}s}{m^2}$$

$$\frac{N\text{-}s}{m^2} = \frac{kg\,m/s^2 \times s}{m^2} = \frac{kg\,m}{m^2\,s} = \frac{kg}{m\text{-}s}$$

From equation ①

$$MLT^{-2} = C\left[L^a \cdot [LT^{-1}]^b \cdot [ML^{-3}]^c \cdot [ML^{-1}T^{-1}]^d\right]$$

$$MLT^{-2} = C\left[L^{a+b-3c-d} \cdot T^{-b-d} \cdot M^{c+d}\right]$$

$M^1 = M^{c+d}$	$L^1 = L^{a+b-3c-d}$	$T^{-2} = T^{-b-d}$
$c+d = 1 \rightarrow ②$	$a+b-3c-d = 1 \rightarrow ③$	$-b-d = -2 \rightarrow ④$

From eq ② $c = 1-d$

from eq ③ $a+b-3c-d = 1 \Rightarrow a+b-3(1-d)-d = 1$

$a+b-3+3d-d = 1 \Rightarrow a+b+2d-3 = 1$

$a+b+2d = 4 \longrightarrow ⑤$

From eq ④ $-b-d = -2 \Rightarrow -b = -2+d \Rightarrow b = 2-d$

From eq ⑤

$$a+b+2d=4 \Rightarrow a+2-d+2d=4$$

$$a+d=4-2$$

$$a=2-d$$

$$MLT^{-2} = c\left[L^{(2-d)} \cdot [LT^{-1}]^{2-d} [ML^{-3}]^{1-d} [ML^{-1}T^{-1}]^{d} \right]$$

$$MLT^{-2} = c\left[L^{2}L^{-d} \; L^{2}L^{-d} \; T^{-2}T^{d} \; M^{1}M^{-d} \; L^{-3}L^{3d} \; M^{d}L^{-d} T^{-d} \right]$$

$$MLT^{-2} = c[L \cdot L^{0} \; T^{-2}T^{0} \; M^{1}M^{0}] = MLT^{-2}$$

$$MLT^{-2} = c\left[(L^{2}L^{-3} \times L^{2}] T^{-2} \times M \times [L^{-d} \times L^{-d} T^{d} M^{-d} L^{3d} M^{d} L^{-d} T^{-d} \right]$$

$$MLT^{-2} = c\left[[L^{2}L^{-3}L^{2}T^{-2} \times M] \times \left[\left(\frac{1}{L} \times \frac{1}{L} \times T \times \frac{1}{M} \times L^{3} \times M \times \frac{1}{L} \times \frac{1}{T}\right)^{d}\right] \right]$$

$$MLT^{-2} = c[D^{a} u^{b} \rho^{c} \mu^{d}]$$

$$MLT^{-2} = c\left[D^{2-d} \; u^{2-d} \; \rho^{1-d} \; \mu^{d} \right]$$

$$F = c\left[D^{2}D^{-d} \; u^{2}u^{-d} \; \rho^{1}\rho^{-d} \; \mu^{d} \right]$$

$$F = c\left[(D^{2}u^{2}\rho) \times D^{-d} \rho^{-d} \mu^{d} u^{-d} \right]$$

$$F = c\left[D^{2}u^{2}\rho \times \left[\frac{\mu}{\rho D u}\right]^{d} \right] \longrightarrow \text{Dimensional parameter reynolds number}$$

$$F = D^{2}u^{2}\rho \; \phi (Re]$$

Buckingham's π theorem

* This is used as a rule of thumb for determining the number of independent dimensionless groups that can be obtained from a set of variables

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* It states that the number of independent dimensionless groups that can be formed from a set of "n" variables having "r" basic dimensions is (n-r)

Basic dimensions are mass, length, time, temperature

mass → M, length → L, time → T, temperature → θ.

let A_1, A_2, A_3 ---- A_6 → six variables

$r = 4, \quad n = 6 \qquad (n-r) = 6-4 = 2$

According to the buckingham π theorem the relation ship b/w there dimensionless groups can be expressed as

$$F(\pi_1, \pi_2) = 0$$

Dimensional Analysis for forced convection

Let us consider the case of a fluid flow across a heated tube

	Symbol	Dimension
① Tube diameter	D	L
② Fluid density	ρ	ML^{-3}
③ Fluid velocity	u	LT^{-1}
④ Fluid viscosity	μ	$ML^{-1}T^{-1}$
⑤ Specific heat	C_p	$L^2T^{-2}\theta^{-1}$
⑥ Thermal conductivity	k	$MLT^{-3}\theta^{-1}$
⑦ heat transfer Coefficient	h	$MT^{-3}\theta^{-1}$

$C_p \rightarrow$

$J/kg\text{-}k$

$\frac{N\text{-}m}{kg\text{-}K}$

$\frac{\frac{kg\text{-}m}{s^2} \times m}{kg\text{-}k}$

$k \rightarrow \frac{W}{m\text{-}k} = \frac{J}{m\text{-}k}$

$\frac{N}{sk} = \frac{\frac{kgm}{s^2}}{sk}$

$\frac{kgm}{s^3 k}$

$h \rightarrow$

$\frac{W}{m^2 k} = \frac{N\text{-}m}{\ldots} = \ldots$

Number of independent dimensionless groups

$(n-r) = (7-4) = 3$

These groups are symbolized as

π_1, π_2, π_3

$h = (D, \mu, C_p, u, k, \rho) \Rightarrow h = F(\pi_1, \pi_2, \pi_3)$

let us consider core variables D, ρ, μ, k

non-core variables are C_p, u, h

$\pi_1 = D^a \rho^b \mu^c k^d u$, $\pi_2 = D^e \rho^f \mu^g k^i C_p$, $\pi_3 = D^j \rho^l \mu^m k^n h$

For π_1 $M^0L^0T^0\theta^0 \longrightarrow = L^a (ML^{-3})^b (ML^{-1}T^{-1})^c [MLT^{-3}\theta^{-1}]^d [LT^{-1}]$

$M^0L^0T^0\theta^0 = M^{b+c+d} L^{a-3b-c+d+1} T^{-c-3d-1} \theta^{-d}$

$M^{b+c+d} = M^0 \Rightarrow b+c+d = 0$

$L^{a-3b-c+d+1} = L^0 \Rightarrow a-3b-c+d+1 = 0$

$T^{-c-3d-1} = T^0 \Rightarrow -c-3d-1 = 0$

$\theta^{-d} = \theta^0 \Rightarrow -d = 0 \Rightarrow d = 0$

if $d = 0 \Rightarrow -c-3(0)-1 = 0 \Rightarrow -c = 1 \Rightarrow c = -1$

$a-3b-c+d+1 = 0 \Rightarrow a-3b-(-1)+0+1 = 0$

$a-3b+2 = 0$

$b+c+d = 0 \Rightarrow b+(-1)+0 = 0 \Rightarrow b = 1$

$a-3(1)+2 = 0 \Rightarrow a = 2$

$a = 1,\ b = 1,\ c = -1,\ d = 0$

$\pi_1 = D^a \rho^a \mu^c k^d u = D^1 \rho^1 \mu^{-1} k^0 u = \frac{D\rho u}{\mu}$

$\pi_1 = \frac{\rho u D}{\mu} \longrightarrow$ Reynolds number $\Rightarrow \pi = Re_p$

$\pi_2 = D^e \rho^f u^g k^i C_p$

$M^0L^0T^0\theta^0 = [L]^e [ML^{-3}]^f [ML^{-1}T^{-1}]^g [MLT^{-3}\theta^{-1}]^i [L^2T^{-2}\theta^{-1}]$

$$M^0L^0T^0\theta^0 = M^{f+g+i} \; L^{e-3f-g+i+2} \; T^{-g-3i-2} \; \theta^{-i-1}$$

$f+g+i=0$, $0=e-3f-g+i+2$, $-g-3i-2=0$, $-i-1=0$
$i=-1$

$\Rightarrow -g-3(-1)-2=0 \Rightarrow -g+3-2=0 \Rightarrow g=1$

$\Rightarrow f+g+i=0 \Rightarrow f+(1)+(-1)=0 \Rightarrow F=0$

$e-3f-g+i+2=0 \Rightarrow e-3(0)-(1)+(-1)+2=0$

$e-0-1-1+2=0 \Rightarrow e=0$

$$\pi_2 = D^e \rho^f \mu^g k^i c_p = D^0 \rho^0 \mu^1 k^{-1} c_p = \frac{c_p \mu}{k}$$

$\pi_2 = \frac{c_p \mu}{k} \rightarrow$ prandtl number (P_r) $(e=0, f=0, g=1, i=-1)$

$\pi_3 = D^j \rho^l \mu^m k^n h$

$$M^0L^0T^0\theta^0 = [L^j][ML^{-3}]^l [ML^{-1}T^{-1}]^m [MLT^{-3}\theta^{-1}]^n [MT^{-3}\theta^{-1}]$$

$$M^0L^0T^0\theta^0 = M^{l+m+n+1} \; L^{j-3l-m+n} \; T^{-m-3n-3} \; \theta^{-n-1}$$

$l+m+n+1=0$ $-m-3n-3=0$

$j-3l-m+n=0$ $-n-1=0 \Rightarrow n=-1$

$\Rightarrow -m-3n-3 = -m-3(-1)-3=0 \Rightarrow -m+3-3=0 \Rightarrow m=0$

$l+m+n+1=0 \Rightarrow l+0-1+1=0 \Rightarrow l=0$

$j-3l-m+n=0 \Rightarrow j-0-0-1=0 \Rightarrow j=1$

$$\pi_3 = D^j \rho^l \mu^m k^n h = D^1 \rho^0 \mu^0 k^{-1} h$$

$\pi_3 = \frac{hD}{k} \rightarrow$ Nusselt number

$f(\pi_1, \pi_2, \pi_3)$ as $Nu = \phi(Re \; Pr)$

$$Re \times P_r = \frac{\rho u D}{\mu} \frac{\mu C_p}{k} = \frac{\rho u C_p D}{k}$$

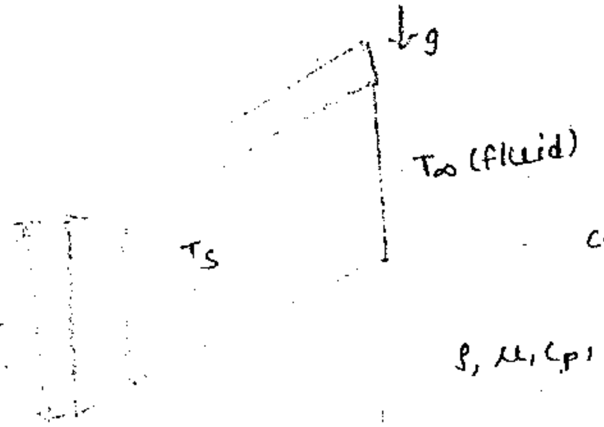
$$\pi_3 = \phi(\pi_1, \pi_2)$$

$$Nu = \frac{hD}{k} \qquad Re\,P_r = \frac{\rho u C_p D}{k}$$

$$h \to MT^{-3}\theta^{-1} \qquad \rho \times u \times C_p = \frac{M}{L^3} \times \frac{L}{T} \times \frac{L^2}{T^2\theta} = MT^{-3}\theta^{-1}$$

$$Nu = Re\,P_r$$

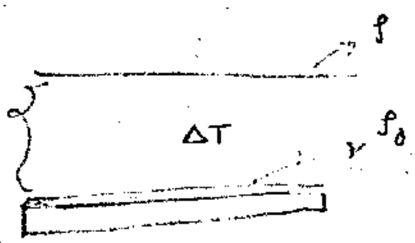
Dimensional Analysis for Free convection (or) Natural convection



coefficient of expansion

$\rho, \mu, C_p, k, (\beta g)$

$\rho \to$ bulk density $\quad \rho_0 \to$ density of the fluid inside the heated layer



$\Delta T \to$ Temperature difference between the heated fluid and the bulk value

$$\rho = (\rho_0 + \rho_0 \beta \Delta T) = \rho_0(1 + \beta\Delta T)$$

$F_B \to$ Buoyant force $\quad F_B = (\rho - \rho_0) g = ((\rho_0 + \rho_0 \beta \Delta T) - \rho_0) g$

$$= \rho_0 \beta \Delta T g \Rightarrow \beta g \rho_0 \Delta T$$

$$w = \frac{W}{V} = \frac{mg}{V} = \rho g$$

variables	symbol	Dimensions
① Fluid density	ρ	ML^{-3}
② Fluid viscosity	μ	$ML^{-1}T^{-1}$
③ Fluid heat capacity	C_p	$L^{2}T^{-2}\theta^{-1}$
④ Fluid thermal conductivity	k	$MLT^{-3}\theta^{-1}$
⑤ Fluid coefficient of thermal expansion	β	θ^{-1}
⑥ gravitational acceleration	g	LT^{-2}
⑦ Temperature difference	ΔT	θ
⑧ Significant length	L	L
⑨ heat transfer coefficient	h	$MT^{-3}\theta^{-1}$

w.r.t $(n-r) = (8-4) = 4$ $\quad$ ($\therefore$ β and g taken together)

let us consider the core variables L, ρ, μ, K

non-core variables are ΔT, βg, C_p, h

$$\pi_1 = L^a \rho^b \mu^c K^d \Delta T \quad , \quad \pi_2 = L^a \rho^b \mu^c K^d \beta g$$

$$\pi_3 = L^a \rho^b \mu^c K^d c_p \quad , \quad \pi_4 = L^a \rho^b \mu^c K^d h$$

$$\pi_1 = M^0 L^0 T^0 \theta^0 = [L]^a [ML^{-3}]^b [ML^{-1}T^{-1}]^c [MLT^{-3}\theta^{-1}]^d [\theta]$$

$$\pi_1 = M^0 L^0 T^0 \theta^0 = M^{b+c+d} L^{a-3b-c+d} T^{-c-3d} \theta^{-d+1}$$

$b+c+d = 0$, $\quad a-3b-c+d = 0$, $\quad -c-3d = 0$; $\quad -d+1 = 0$

$b+(-3)+1 = 0$ $\qquad a-3(2)-(-3)+1 = 0$ $\qquad -c-3(1) = 0$ $\qquad d = 1$

$b = 2$ $\qquad a = 2$ $\qquad c = -3$

$$\pi_1 = L^a \rho^b \mu^c K^d \,\Delta T = [L]^2 [\rho]^2 [\mu]^{-3} [K]^1 \Delta T$$

$$\pi_1 = L^2 \rho^2 \mu^{-3} K \Delta T = \frac{L^2 \rho^2 K \Delta T}{\mu^3}$$

$$\pi_2 = L^a \rho^b \mu^c K^d \,\beta g$$

$$M^0 L^0 T^0 \Theta^0 = [L]^a [ML^{-3}]^b [ML^{-1}T^{-1}]^c [MLT^{-3}\theta^{-1}]^d \,\theta^{-1} L T^{-2}$$

$$M^0 L^0 T^0 \Theta^0 = M^{b+c+d} \, L^{a-3b+d-c+1} \, T^{-c-3d-2} \, \theta^{-d-1}$$

$$b+c+d = 0 , \quad a-3b-c+d+1 = 0 \quad -c-3d-2=0 \quad -d-1=0$$

$$-d = 1$$

$$-c-3(-1)-2=0 \quad d = -1$$

$$b+1-1 = 0 \quad a-3(0)-1-1+1=0 \quad -c+3-2=0$$

$$b = 0 \quad a-2+1=0 \quad c = -1$$

$$a-1=0$$

$$a = 1$$

$$\pi_2 = L^a \rho^b \mu^c K^d \beta g = L^1 \rho^0 \mu^1 K^{-1} \beta g = \frac{L \mu \beta g}{K}$$

$$\pi_1 \pi_2 = \frac{L^2 \rho^2 K \Delta T}{\mu^3} \times \frac{L \mu \beta g}{K} = \frac{\rho^2 L^3 \beta \Delta T g}{\mu^2}$$

$$G_r = (\pi_1 \pi_2) = \frac{\rho^2 L^3 \beta \Delta T g}{\mu^2}$$

$$\pi_3 = L^a \rho^b \mu^c K^d \, C_p$$

$$M^0 L^0 T^0 \Theta^0 = [L]^a [ML^{-3}]^b [ML^{-1}T^{-1}]^c [MLT^{-3}\theta^{-1}]^d \, L^2 T^{-2} \theta^{-1}$$

$$M^0 L^0 T^0 \Theta^0 = M^{b+c+d} \, L^{a-3b-c+d+2} \, T^{-c-3d-2} \, \theta^{-d-1}$$

$$a-3b-c+d+2=0 \quad -c-3d-2=0 \quad -d-1=0$$

$$b+c+d=0 \quad d=-1$$

$$b+1-1=0 \quad a-3(0)-1-1+2=0 \quad -c-3(-1)-2=0$$

$$-c+3-2=0$$

$$b=0 \quad a-2+2=0$$

$$c = 1$$

$$a = 0$$

$$\pi_3 = L^a \rho^b \mu^c k^d C_p = L^0 \rho^0 \mu^1 k^{-1} C_p = \frac{\mu C_p}{k}$$

$$\pi_3 = \frac{\mu C_p}{k} \rightarrow \text{prandtl number } (P_r)$$

$$\pi_4 = L^a \rho^b \mu^c k^d h$$

$$M^0L^0T^0\theta^0 = [L]^a \, [ML^{-3}]^b \, [ML^{-1}T^{-1}]^c \, [MLT^{-3}\theta^{-1}]^d \, [MT^{-3}\theta^{-1}]$$

$$M^0L^0T^0\theta^0 = M^{b+c+d+1} \, L^{a-3b-c+d} \, T^{-c-3d-3} \, \theta^{-d-1}$$

$b+c+d+1=0$ | $a-3b-c+d=0$ | $-c-3d-3=0$ | $-d-1=0$, $d=-1$

$b+0-1+1=0$ | $a-3(0)-0-1=0$ | $-c-3(-1)-3=0$

$b=0$ | $a=1$ | $-c=0$, $c=0$

$$\pi_4 = L^a \rho^b \mu^c k^d h = L^1 \rho^0 \mu^0 k^{-1} h = \frac{Lh}{k}$$

$$\pi_4 = \frac{hL}{k} \rightarrow \text{Nusselt number}$$

$$\pi_4 = \frac{hL}{k} \rightarrow \text{dimensionless number}$$

$$G_r = \pi_1 \pi_2 = \frac{\rho^2 \beta g L^3 \Delta T}{\mu^2}, \quad P_r = \pi_3 = \frac{\mu C_p}{k}$$

$$G_r \cdot P_r = \frac{\rho^2 \beta g L^3 \Delta T}{\mu^2} \times \frac{\mu C_p}{k} = \frac{\rho^2 \beta g L^2 \Delta T C_p \times \mu}{\mu^2} \times \frac{L}{k}$$

$$= \frac{[ML^{-3}]^2 \times \theta^{-1} \times LT^{-2} \times L^2\theta \, L^2T^{-2}\theta^{-1}}{ML^{-1}T^{-1}}$$

$$= \frac{M^2L^{-6} \times \theta^{-1} \, LT^{-2} \times L^2 \times \theta L^2 T^{-2} \times \theta^{-1}}{ML^{-1}T^{-1}}$$

$$= \frac{M \cdot L^2 \cdot L^2 \times L^2}{L^6 \cdot T^2 \cdot T \cdot T^2} \times \frac{1}{\theta} \times LT = \frac{M}{T^3\theta}$$

$$= MT^{-3}\theta^{-1}$$

$h = MT^{-3}\theta^{-1}$

$Nu = \phi(Gr \cdot Pr) \longrightarrow$ free convection

$Nu = \phi(Re\ Pr) \longrightarrow$ Forced convection

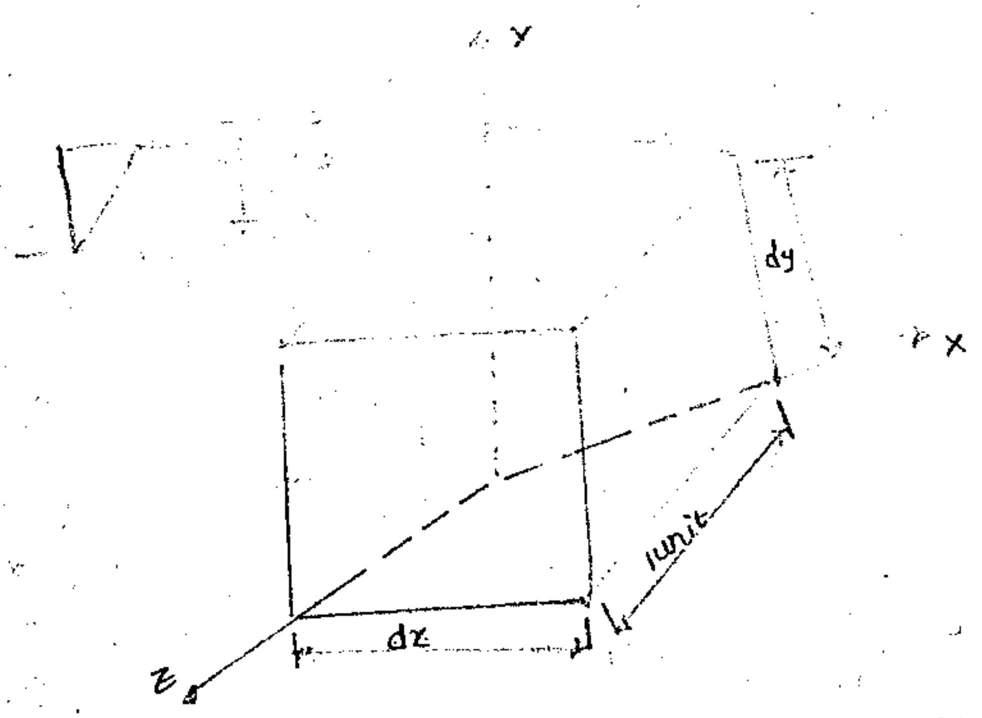
momentum equations

→ momentum equation is derived from Newton's II law of motion

* This law requires sum of all the forces acting from the control volume must be equal to the rate of increase of the fluid momentum

$F = ma \Rightarrow \Sigma F = ma$

Forces acting on the fluid → ① Body force
② surface force



mass × acceleration = Body force + surface force

$\text{mass} = \rho \times dx\ dy\ 1 = \rho\, dx\, dy$

In order to find the acceleration use the concept of substantial derivative consider ϕ as a function (x, y, t)

$\phi \rightarrow$ may be velocity component, pressure (or) temperature of fluid element

let $d\phi$ is the differential change of function ϕ where the fluid moves from (x, y) to $(x+dx, y+dy)$

$$d\phi = \frac{\partial \phi}{\partial t} \times dt + \frac{\partial \phi}{\partial x} dx + \frac{\partial \phi}{\partial y} dy$$

$$\frac{d\phi}{dt} = \frac{\partial \phi}{\partial t} + \frac{\partial \phi}{\partial x} \frac{dx}{dt} + \frac{\partial \phi}{\partial y} \frac{dy}{dt}$$

$$\underset{\text{Global function}}{\frac{D\phi}{Dt}} = \underset{\text{local function}}{\frac{\partial \phi}{\partial t}} + \frac{\partial \phi}{\partial x} u + \frac{\partial \phi}{\partial y} v$$

$\frac{\partial \phi}{\partial t} \rightarrow$ local rate of change

$$\frac{D\phi}{Dt} = u \frac{\partial \phi}{\partial x} + v \frac{\partial \phi}{\partial y}$$

$\phi \rightarrow u$ and v

$$\underbrace{\frac{Du}{Dt} = u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \rightarrow (1), \quad \frac{Dv}{Dt} = u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \rightarrow (2)}_{\text{acceleration component}}$$

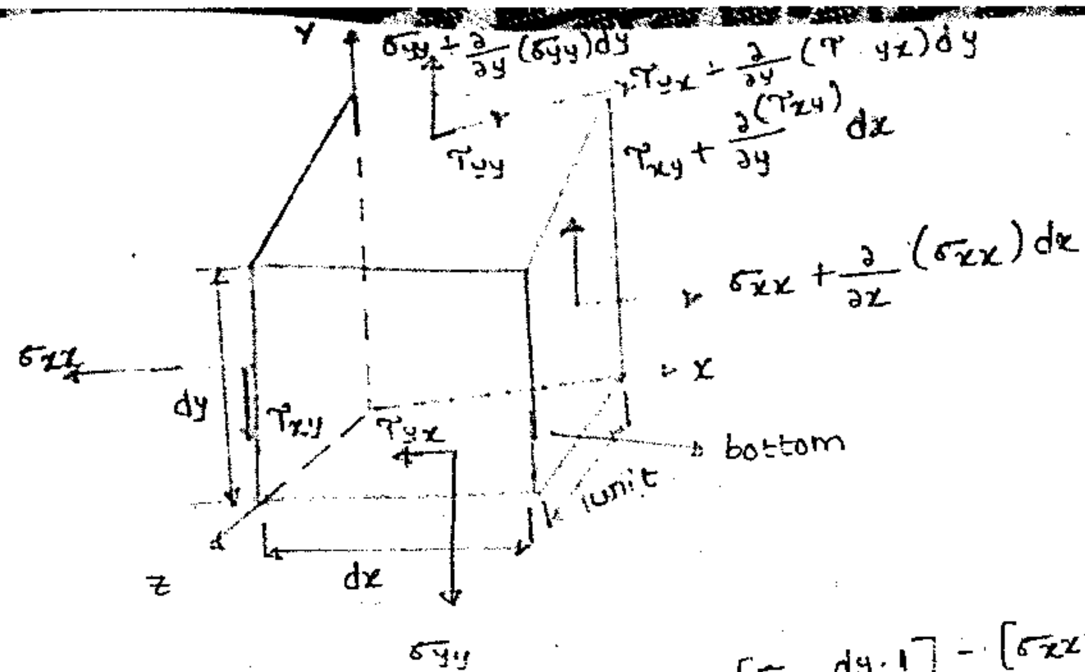
<u>Body force terms</u>

B_x and B_y be the body forces per unit volume in x and y directions respectively

Body force in x-direction $= B_x (dx\, dy \cdot 1)$

Body force in y-direction $= B_y (dx\, dy \cdot 1)$

<u>Surface forces</u>



Net force acting in x-direction $= [\sigma_{xx}\, dy \cdot 1] - [\sigma_{xx} + \frac{\partial}{\partial x}(\sigma_{xx})\, dx]\times dy \times 1$

$+ \tau_{yx}\, dx(1) - [\tau_{yx} + \frac{\partial}{\partial y}(\tau_{yx})\, dy]\, dx \cdot 1$

$$= \frac{\partial}{\partial x}(\sigma_{xx})\, dx\, dy \cdot 1 + \frac{\partial}{\partial y}(\tau_{yx})\, dx\, dy \cdot 1$$

x-direction $= \frac{\partial}{\partial x}\sigma_{xx}\, dx\, dy + \frac{\partial \tau_{yx}}{\partial y}\, dx\, dy$

Net force acting in y-direction $= \frac{\partial \sigma_{yy}}{\partial y}\, dx\, dy + \frac{\partial \tau_{xy}}{\partial x}\, dx\, dy$

Therefore momentum in x-direction

$$\rho\, dx\, dy\left[u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right] = B_x\, dx\, dy + \frac{\partial \sigma_{xx}}{\partial x}\, dx * dy + \frac{\partial \tau_{yx}}{\partial y}\, dx\, dy$$

$$\rho\left[u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right] = B_x + \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y}$$

x-momentum direction

$$\rho\left[u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right]=B_y+\frac{\partial\sigma_{yy}}{\partial y}+\frac{\partial\tau_{xy}}{\partial x}$$

$\longrightarrow$ y-momentum equation

① Water flows in a duct having a cross section 5x10mm with a mean bulk temperature of 20°C. If the duct wall temperature is constant at 60°C and fully developed laminar flow is experienced, calculate the heat transfer per unit length

Sol:- given that

T=20°c

water flows $\longrightarrow$ 60°c

10 mm

$Q = hA\,\Delta T$

(i) mean bulk temperature

(ii) Fully developed laminar flow

(iii) constant wall temperature

$$\frac{Q}{L}=?\,,\quad D_h=\frac{4A}{P}=\frac{4\times5\times10\times10^{-6}}{2(5+10)\times10^{-3}}=\frac{20}{3}\times10^{-3}$$

Let us assume duct is larger in length $L/D_h > 100$

$$\frac{2b}{2a}=\frac{5}{10}=\frac{1}{2}$$

For fully developed laminar flow (i.e constant wall temperature) $Nu = 3.391$

$$Nu=\frac{hL}{K}\text{ or }\frac{hD_h}{K}\qquad Nu=\frac{\text{temperature gradient by conduction}}{\text{convection at the surface}}$$

$$Nu=\frac{KA\frac{\Delta T}{\Delta x}}{hA\,\Delta T}=\frac{k}{hL}$$

$$Q = kA\frac{\Delta T_c}{L}, \quad Q = hA\Delta T_{cv}$$

$$\Delta T_c = \frac{Q \times L}{kA}, \quad \Delta T_{cv} = \frac{Q}{hA}$$

$$\Delta T_c = \frac{\Delta T_{cv} \times hA \times L}{kA}, \quad \frac{\Delta T_c}{\Delta T_{cv}} = \frac{hL}{k} = Nu$$

$$Nu = 3.391 = \frac{hL}{k} = \frac{\Delta T_{cond}}{\Delta T_{convec}} = \frac{\Delta T_{cond}}{(Q/hA)}$$

$$Q = hA(T_s - T_f) \Rightarrow \frac{Q}{L} = \frac{hA}{L}(T_s - T_f)$$

$$Nu = 3.391 = \frac{hL}{k} \Rightarrow h = \frac{3.391k}{L} = \frac{\Delta T_{cond}}{\Delta T_{conv}}$$

$$h = 3.391 \times \left(\frac{\Delta T_{conv}}{L}\right) = \left[\frac{\Delta T_{cond}}{k}\right]$$

For fully devloped laminar flow $Nu = 3.391$

$$Nu = \frac{hL}{k} \Rightarrow 3.391 = \frac{h \times 1}{k} \Rightarrow h = 3.391\,k \qquad \underline{\text{at } 20^\circ}$$

For water $k = 0.5978\ W/m\text{-}K$, $\rho = 1000\ kg/m^3$, kinematic viscosity $\nu = 1.006 \times 10^{-6}\ m^2/s$, thermal diffusitivity $\alpha = 0.1431 \times 10^{-6}\ m^2/s$, specific heat $C_p = 4178\ J/kg\text{-}K$, prandtl number $Pr = 7.020$,

$$h = 3.391 \times 0.5978 = 2.027\ W/m^2K$$

$$Q = hA\Delta T = hA(T_s - T_f) \Rightarrow \frac{Q}{L} = hA\Delta T \rightarrow \text{unit length}$$

$$\bar{q} = \frac{Q}{L} = \frac{h}{L}A(T_s - T_f) = hA(T_s - T_f)$$

$$\bar{q} = 2.027 \times 2 \times 15 \times (60 - 20) \times 10^{-3}$$

$$\bar{q} = 2.4324\ W/m$$

$A = 2(5 \times 1) + 10 \times 1$ $L = 1$

$= 2(15) = 30\,mm^2$

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$Nu = Re\,Pr \longrightarrow$ Forced convection

$$\frac{Nu}{Pr} = Re \Rightarrow Re = \frac{3.391}{7.020} = 0.483$$

$$Re = \frac{\rho u \times L}{\mu} = \frac{u \times L}{\mu/\rho} = \frac{u \times L}{\nu}$$

$$\mu = \rho \times \nu$$

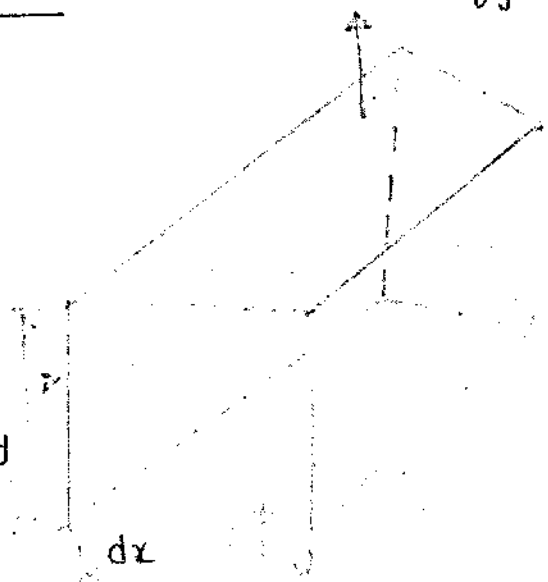
$$\frac{\mu}{\rho} = \nu$$

$$0.483 \times 1.006 \times 10^{-6}\ m^2/s = u \times 1\,(m)$$

$$u = 0.483 \times 1.006 \times 10^{-6}\ m/s = 0.485 \times 10^{-6}\ m/s$$

.... heated tube

Energy equation



According to the first law of thermodynamics

$$\frac{dQ}{dt} = \frac{dE}{dt} + \frac{dW}{dt}$$

heat transfer in to the element is done by the mode of conduction where in the absence of radiation

$$\frac{dQ}{dt} = -\left[\frac{\partial Q_x}{\partial x} dx + \frac{\partial Q_y}{\partial y} dy\right] \qquad \left[Q_x = -KA\frac{\partial T}{\partial x}\right]$$

$$\frac{dQ}{dt} = -\left[\frac{\partial}{\partial x}\left(-KA\frac{\partial T}{\partial x}\right)dx + \frac{\partial}{\partial y}\left(-KA\frac{\partial T}{\partial y}\right)dy\right]$$

$$\frac{dQ}{dt} = K\left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right] dx\,dy$$

The internal energy comprises the specific internal energy "e" per unit mass

Kinetic energy $= \frac{1}{2}(u^2 + v^2)$ per unit mass

$$E = J = N\text{-}m \rightarrow \frac{kg\text{-}m \times m}{s^2} \rightarrow kg\left[\frac{m}{s}\right]^2$$

$$\frac{dE}{dt} = \rho\left[\frac{De}{Dt} + \frac{1}{2}\frac{D}{Dt}(u^2+v^2)\right] dx\,dy.1$$

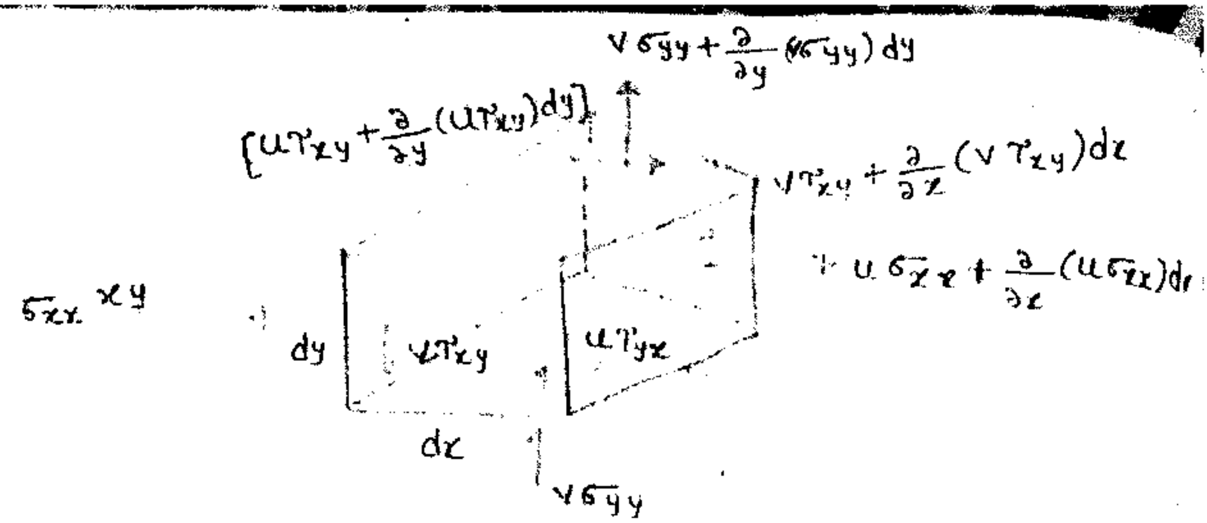
(i) Body force (ii) surface force & frictional force

let B_x and B_y are the body forces acting per unit volume of the fluid in the x and y directions respectively

then the rate of workdone by the Body forces

$$\frac{dW_B}{dt} = (B_x u + B_y v)\,dx\,dy.1$$

rate of work done due to normal stress in shear stress



rate of workdone perfomed on the element due to the normal stress σ_{xx} is given by

$$\left[-u\sigma_{xx} + u\sigma_{xx} + \frac{\partial}{\partial x}(u\sigma_{xx})\,dx\right]dy = \frac{\partial}{\partial x}(u\sigma_{xx})\,dx\,dy$$

Similarly in the y-direction due to normal stresses

$$\frac{\partial}{\partial y}(v\sigma_{yy})\,dx\,dy$$

The rate of work done due to shear stress τ_{xy} and τ_{yx}

$$\frac{\partial}{\partial x}(v\tau_{xy})\,dx\,dy \quad \text{and} \quad \frac{\partial}{\partial y}(u\tau_{yx})\,dx\,dy$$

Thus the total rate of work done due to friction is

$$\frac{dw_f}{dt} = \left[\frac{\partial}{\partial x}(u\sigma_{xx})\,dx\,dy + \frac{\partial}{\partial x}(v\tau_{xy})\,dx\,dy\right] + \left[\frac{\partial}{\partial y}(v\sigma_{yy})\,dx\,dy + \frac{\partial}{\partial y}(u\tau_{yx})\,dx\,dy\right]$$

$$\frac{dw_f}{dt} = \left[\frac{\partial}{\partial x}(u\sigma_{xx} + v\tau_{xy})\,dx\,dy\right] + \left[\frac{\partial}{\partial y}(v\sigma_{yy} + u\tau_{yx})\,dx\,dy\right]$$

Total rate of workdone due to friction and body force

$$\frac{dw}{dt} = \frac{-dw_f}{dt} - \frac{dw_B}{dt}$$

$$= -\left[uB_x + vB_y + \frac{\partial}{\partial x}(u\sigma_{xx} + v\tau_{xy}) + \frac{\partial}{\partial x}(v\sigma_{yy} + u\tau_{yx})\right] dx\,dy$$

Substituting $\frac{dQ}{dt}$, $\frac{dE}{dt}$ in $\frac{dW}{dt}$ we get

eq^n $\frac{dQ}{dt} = \frac{dE}{dt} + \frac{dW}{dt}$

$$\frac{dQ}{dt} = \rho\frac{De}{Dt} + \frac{\rho}{2}\frac{D}{Dt}(u^2+v^2) - \left[uB_x + vB_y + \frac{\partial}{\partial x}(u\sigma_{xx} + v\tau_{xy}) + \frac{\partial}{\partial x}(v\sigma_{yy} + \tau_{yx})\right] dx\,dy$$

$$\rho\frac{De}{Dt} + \frac{\rho}{2}\frac{D}{Dt}(u^2+v^2) = k\left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right] + \left[uB_x + vB_y + \frac{\partial}{\partial x}(u\sigma_{xx}) + \frac{\partial}{\partial y}(v\sigma_{yy}) + \frac{\partial}{\partial y}(u\tau_{yx}) + \frac{\partial}{\partial x}(v\tau_{xy})\right] \longrightarrow ①$$

W.K.T x-momentum equation

$$\rho\left[u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right] = B_x + \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial x} \longrightarrow ②$$

$$\rho\left(u^2\frac{\partial u}{\partial x} + uv\frac{\partial u}{\partial y}\right] = uB_x + u\frac{\partial \sigma_{xx}}{\partial x} + \frac{u\,\partial}{\partial x}(\tau_{yx})$$

$$\rho\left[u^2\frac{\partial u}{\partial x} + uv\frac{\partial u}{\partial y}\right] = uB_x + \frac{\partial}{\partial x}(u\sigma_{xx}) + \frac{\partial}{\partial x}(u\tau_{yx})$$

eq ① can now be simplified by using the momentum equation ② is multiplied by u and eq ③ is multiplied by v and adding these we get

y-momentum equation

$$\rho\left[u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right] = B_y + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \tau_{xy}}{\partial y} \longrightarrow ③$$

$$\frac{\rho}{2}\frac{D}{Dt}(u^2+v^2) = \left[uB_x + vB_y + u\frac{\partial\sigma_{xx}}{\partial x} + v\frac{\partial\sigma_{yy}}{\partial y} + u\frac{\partial\tau_{yx}}{\partial y} + v\frac{\partial\tau_{xy}}{\partial x}\right] \longrightarrow (4)$$

since

$$u\frac{Du}{Dt} = \frac{1}{2}\frac{Du^2}{Dt} \quad \text{and} \quad \frac{vDv}{Dt} = \frac{1}{2}\frac{Dv^2}{Dt}$$

substracting eq (1) and (4) we get

$$\rho\frac{De}{Dt} = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right] + \left[\sigma_{xx}\frac{\partial u}{\partial x} + \sigma_{yy}\frac{\partial v}{\partial y} + \tau_{yx}\frac{\partial u}{\partial y} + \tau_{xy}\frac{\partial v}{\partial x}\right] \longrightarrow (5)$$

since various stresses in eq (5)

$$\rho\frac{De}{Dt} = k\left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right] + \mu\left(2\left(\frac{\partial u}{\partial x}\right)^2 + 2\left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2\right) \longrightarrow (6)$$

The equation (energy) (6) is written in a compact form as

$$\rho\frac{De}{Dt} = k\left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right] + \mu\phi \longrightarrow (7)$$

where the viscous energy - dissipation term ϕ is defined as

$$\phi = 2\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2\right] + \left[\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right]^2$$

For an incompressible fluid we may written the term $\frac{De}{Dt}$ as

$$\frac{De}{Dt} = c_p\frac{DT}{Dt}$$

Thus the equation energy eq (7) $\rho c_p\left[u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right] = k\left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right] + \mu\phi$

For many flows in applications the flow velocities are not really large hence the viscous energy dissipation is small and can be neglected

$$\rho c_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = k\left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right] \longrightarrow (8)$$

Expressed in vector notation $\rho c_p(V\nabla T) = k\nabla^2 T$

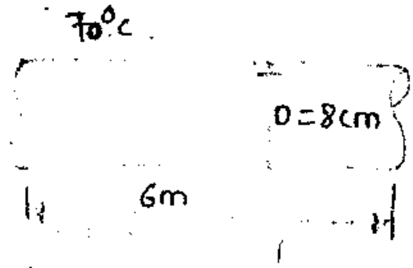
In cyclindrical polar coordinates the energy equation takes the form

$$\rho c_p\left[v_r\frac{\partial T}{\partial x} + u\frac{\partial T}{\partial r}\right] = k\left[\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial x^2}\right]$$

where u and v_r are the velocities in x and r direction respectively

① A 6m long section of an 8cm diameter horizontal hot water pipe shown in the Fig. passes through a large room whose temperature is 20°c. If the outer surface temperature of the pipe is 70°c Determine the rate of heat loss from the pipe by natural convection

Sol:- given that $T_\infty = 20°c$



Heat loss $Q = hA\Delta T = h \times A(T_s - T_\infty)$

$$= h \times 2\pi \times r \times L\,(T_s - T_\infty)$$

$Nu = G_r \times P_r$ → Natural (or) free convection

$Nu = R_e \times P_r$ → Forced convection

W.K.T

Grasheff number

$$G_r = \frac{g\beta\Delta T L^3 \rho^2}{\mu^2}$$

Prandtl number

$$P_r = \frac{C_p \mu}{k}$$

heat carrier is air

$$Nu = \frac{hL}{k} = \frac{g\beta\Delta T L^3 \rho^2}{\mu^2} \times \frac{C_p \mu}{k}$$

$$h = \frac{g\beta\Delta T\ L^2 \times \rho^2 C_p}{\mu}$$

Temperature of air at $T = \frac{T_s + T_a}{2} = \frac{70+20}{2} = 45°c$

coefficient of expansion of the air $\beta = 3400 \times 10^{-6}/°c$ @ atmospheric pressure at 20°c

$\Delta T = (70-20) = 50°c$, $L = 6m$

ρ at 45°C , μ at 45°C

$\rho_{40°c} = 1.128\ kg/m^3$, $\rho_{50°c} = 1.093\ kg/m^3$

$\mu_{40°c} = 19.12\times10^{-6}\ N\text{-}s/m^2$, $\mu_{50°c} = 19.61\times10^{-6}\ N\text{-}s/m^2$

$$\frac{50°c - 45°c}{50°c - 40°c} = \frac{\rho_{50°c} - \rho_{45°c}}{\rho_{50°c} - \rho_{40°c}}$$

$$\frac{5}{10} = \frac{1.093 - \rho_{45°c}}{1.093 - 1.128} \Rightarrow 5\times(1.093 - 1.128) = 10(1.093 - \rho_{45°c})$$

$$\rho = 1.093 - \frac{5\times(1.093 - 1.128)}{10} = 1.093 - (-0.0175)$$

$\rho = 1.1105\ kg/m^3$ at 45°c

$$\frac{50-45}{50-40} = \frac{19.61\times10^{-6} - \mu}{19.61\times10^{-6} - 19.12\times10^{-6}}$$

$$\frac{5}{10}\times(19.61 - 19.12)\times10^{-6} = 19.61\times10^{-6} - \mu$$

$$\mu = [19.61\times10^{-6} - 0.5(19.61 - 19.12)\times10^{-6}]$$

$\mu = 19.365\times10^{-6}$ at 45°c

$$G_r = \frac{g\beta\Delta T L^3 \rho^2}{\mu^2} = \frac{9.81\times3400\times10^{-6}\times50\times6^3\times(1.1105)^2}{(19.365\times10^{-6})^2}$$

$G_r = 1.1846\times10^{12}$

$$P_r = \frac{\mu c_p}{k} =$$

$P_{r\,50°c} = 0.698$, $P_{r\,40°c} = 0.669$

$c_p = 1005\ J/kg\text{-}k$ same at (50°c, 40°c and 45°c)

$k_{50} = 0.02826\ W/m\text{-}k \qquad k_{40} = 0.02756\ W/m\text{-}k$

$$\frac{5}{10} = \frac{0.698 - P_r}{0.698 - 0.669} \Rightarrow 0.0145 = 0.698 - P_r$$

$$P_r = 0.698 - 0.0145$$

$$= 0.6835 \text{ at } 45^\circ c$$

$$\frac{5}{10} = \frac{0.02826 - k}{0.02826 - 0.02756} \Rightarrow 3.5\times10^{-4} = 0.02826 - k$$

$$k = 0.02791\ W/m\text{-}k$$

at $45^\circ c$

$$G_r P_r = 1.1846\times10^{12} \times 0.6835 = 8.0967441\times10^{11}$$

$$G_r P_r = 8.097\times10^{11}$$

$G_r P_r < 10^9 \rightarrow$ laminar flow

$G_r P_r > 10^9 \rightarrow$ Turbulent flow

$$G_{r_D} = \frac{g\beta\Delta T D^3 \rho^2}{\mu^2} = \frac{9.81\times3400\times10^{-6}\times 50\times(8\times10^{-3})^3\times(1.105)^2}{(19.365\times10^{-6})^2}$$

$G_{r_D} = 358994.2594 = 3.589\times10^5$, $P_r = 0.6835$

$G_{r_D} P_r = 3.589\times10^5 \times 0.6835 = 239840.15 = 2.2984\times10^5$

Assume constant wall temperature

$$Nu = c\left[G_{r_D} P_r\right]^m \qquad G_{r_D} P_r \Rightarrow 10^4 \text{ to } 10^7 \rightarrow$$

$c = 0.48$, $m = 0.25$

$$Nu = 0.48\left[2.3984\times10^5\right]^{0.25} = 10.622$$

W.K.T $Nu = \frac{hL}{k}$ or $\frac{hD}{k} \Rightarrow \frac{h\times8\times10^{-2}}{0.02791} = 10.622$

$$h = \frac{10.622\times0.02791}{8\times10^{-2}} = 3.705\ W/m^2K$$

Heat loss $\Rightarrow Q = hA\Delta T = h \times 2\pi \times r \times L \times (70-20)$

$$Q = 3.7057 \times 2 \times \pi \times \frac{8 \times 10^{-2}}{2} \times 6 \times 50$$

$$Q = 3.7057 \times 2 \times \pi \times 4 \times 10^{-2} \times 6 \times 50$$

$$\boxed{Q = 279.403 \text{ W}}$$

Heat Exchangers - 5

Direct contact heat exchanger

Two fluids exchange heat by coming into direct contact

Eg:- open feed water heaters, desuperheaters and Jet condensers

* Recuperators :- surface heat exchanger
In which fluids are separated by a wall

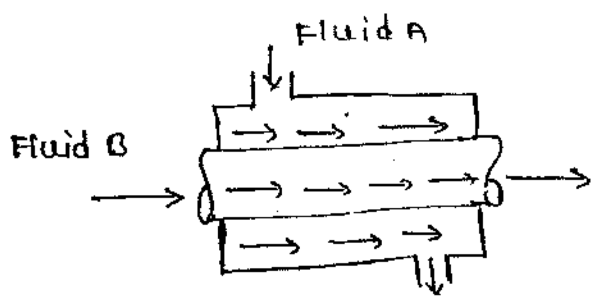
* periodic flow type heat exchanger : regenerator

Eg:- preheators of steam power plant , blast furnaces, oxygen producers

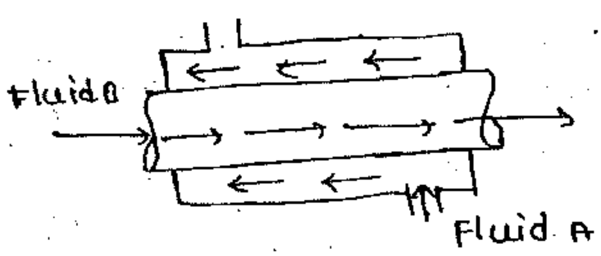
Types of heat exchangers classified based upon the fluid flow

① parallel flow heat exchanger ② Counter flow heat exchanger
③ cross flow heat exchanger ④ shell and tube type heat exchanger

① parallel flow heat exchanger



② counter flow heat exchanger



③ Crossed flow heat exchanger

local heat transfer coefficient $h_x = \frac{k_e}{\delta_x}$

$$h_x = \frac{0.6848 \ W/m\text{-}k}{9.01 \times 10^{-5} \ m} = 7600.44 \ \frac{W}{m^2\text{-}K}$$

$$T_f = \frac{127+117}{2} = 122\,^\circ c$$

liquid $T_f = 122\,^\circ c$

To find the thickness of the film

$$\delta_x = \left[\frac{4\,\mu_l\,k\,x\,(T_v - T_s)}{g\,h_{fg}\times \rho_l\,(\rho_l - \rho_v)}\right]^{0.5}$$

$\rho_v \rightarrow$ very low

$\rho_v = 0$, $\rho_l >>> \rho_v$

given that $x = 0.2\,m$, $T_v = 127\,^\circ c$, $T_s = 117\,^\circ c$

$$\delta_x = \left[\frac{4\,\mu_l\,k\,x\,(T_v - T_s)}{g\,h_{fg}\times \rho_l^2}\right]^{0.5}$$

properties value of liquid has to be taken at $122\,^\circ c$

$$\frac{\rho_1 - \rho_l}{\rho_1 - \rho_2} = \frac{T_1 - T}{T_1 - T_2} \Rightarrow \frac{945 - \rho_l}{945 - 928} = \frac{120-122}{120-140} = 0.1$$

$$\rho_l = -0.1(945 - 928) + 945$$

$$\rho_l = 943.3\ kg/m^3$$

$$\frac{\nu_1 - \nu_l}{\nu_1 - \nu_2} = 0.1 \Rightarrow \frac{0.247 - \nu_l}{0.247 - 0.213} = 0.1 \Rightarrow \nu_l = 0.2426\times 10^{-6}\ m^2/s$$

$$\mu_l = \rho_l \times \nu_l = 943.3 \times 0.2426 \times 10^{-6} = 2.298\times 10^{-4}\ \frac{N}{m\text{-}s}$$

$$\frac{k_1 - k_l}{k_1 - k_2} = 0.1 \Rightarrow \frac{0.6850 - k_l}{0.6850 - 0.6838} = 0.1$$

$$k_l = 0.6848\ W/m\text{-}k$$

$$\delta_x = \left[\frac{4\times 0.6848 \times 2.298\times 10^{-4}\times (127 - 117\,^\circ c)}{9.81\times (943.3)^2\times (2183\times 10^3)}\right]^{1/4}$$

$$\delta_x = 9.01\times 10^{-5}\,m = 0.09\,mm$$

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$$\frac{\rho_1 - \rho}{\rho_1 - \rho_2} = \frac{100-125}{100-200} = \frac{25}{100} = 1/4$$

$$\frac{0.598 - \rho_v}{0.598 - 0.464} = \frac{1}{4} \Rightarrow \rho_v = 0.5645 \ m^3/kg$$

$$\frac{Q}{A} = \rho_l \times \nu_l \times h_{fg} \left[\frac{g(\rho_l - \rho_v)}{\sigma}\right]^{0.5} \left[\frac{C_l \Delta T}{C_{sf} h_{fg} P_r^n}\right]^3$$

$$\frac{Q}{A} = 940.75 \times 0.2385 \times 10^{-6} \times 2185.05 \times 10^3 \left[\frac{9.81(940.75 - 0.5645)}{0.0538}\right]^{0.5} \times$$

$$\left[\frac{4258.25 \times (150-125)}{0.0060 \times 2185.05 \times 10^3 \times 1.394}\right]^3$$

$$\frac{Q}{A} = 0.49024 \times 10^3 \times 414.040 \times [illegible] = 40.05 \times 10^6 \frac{W}{m^2}$$

① Dry saturated steam at a pressure of 2.45 bar condenses on the surface of vertical tube of height 1m. The tube surface temperature is kept at 117°c. Estimate the thickness of the condensate film and the local heat transfer coefficient at a distance of 0.2m from the upper end of the tube

Sol: given that

pressure of dry saturated steam = 2.45 bar

at 1.985 bar T = 120 $\Rightarrow \frac{1.985 - 2.45}{1.985 - 2.701} = \frac{120-T}{120-130}$

at 2.701 bar T = 130

$$0.649 = \frac{120-T}{120-130} \Rightarrow T = 126.49°c \simeq 127°c$$

saturated vapour at T = 127°c

properties of fluid taken at $T_f = \frac{T_{sat} + T_a}{2}$

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$$\frac{Q}{A} = (\rho_l \times \nu_l) \times h_{fg} \times \left[\frac{g(\rho_l - \rho_v)}{\sigma}\right]^{0.5} \left[\frac{C_l \Delta T}{C_{sf}\, h_{fg}\, P_r^{n}}\right]^3$$

$$\frac{\nu_1 - \nu_l}{\nu_1 - \nu_2} = \frac{1}{4} \Rightarrow \frac{0.247 - \nu_l}{0.247 - 0.213} = \frac{1}{4}$$

$$\nu_l = 0.2385 \times 10^{-6}\ m^2/s$$

$$\frac{C_1 - C_l}{C_1 - C_2} = \frac{1}{4} \Rightarrow \frac{4250 - C_l}{4250 - 4283} = \frac{1}{4}$$

$$C_l = 4258.25\ J/kg\text{-}K$$

$$\frac{P_{r_1} - P_{r_L}}{P_{r_1} - P_{r_2}} = \frac{1}{4} \Rightarrow \frac{1.446 - P_{r_L}}{1.446 - 1.241} = 1/4$$

$$P_{r_L} = 1.39475$$

$C_{sf} \to$ brass + water $= 0.0060$

$$\frac{h_{fg_1} - h_{fg}}{h_{fg_1} - h_{fg_2}} = \frac{100 - 125}{100 - 150} = \frac{-25}{-50} = \frac{1}{2}$$

latent heat of vaporisation for water

$$\frac{2256.9 - h_{fg}}{2256.9 - 2113.2} = \frac{1}{2} \Rightarrow h_{fg} = 2185.05\ kJ/kg \text{ at } 125^\circ C$$

σ at 125°C

$$\frac{\sigma_1 - \sigma}{\sigma_1 - \sigma_2} = \frac{100 - 125}{100 - 150} = \frac{1}{2}$$

$$\frac{0.0589 - \sigma}{0.0589 - 0.0487} = \frac{1}{2} \Rightarrow \sigma = 0.0538$$

ρ_v at 125°C

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of 125°c. what is the heat transfer per unit area also

Calculate heat transfer coefficient in boiling.

Sol:- given that

surface temperature $T_s = 150°c$, saturation Temperature

$T_{sat} = 125°c$

property values of liquid (water)

at 120°c $\rho_1 = 945\ kg/m^3$, $\nu_1 = 0.247 \times 10^{-6}\ m^2/s$, $C_{p_1} = 4250\ J/kg\text{-}k$

$P_{r_1} = 1.446$ $\qquad k_1 = 0.6850\ W/m\text{-}k$

at 140°c $\rho_2 = 928\ kg/m^3$ $\quad \nu_2 = 0.213 \times 10^{-6}\ m^2/s$, $\quad P_{r_2} = 1.241$

$C_{p_2} = 4283\ J/kg\text{-}k$, $k_2 = 0.6838\ \frac{W}{m\text{-}k}$

at 125°c → interpolation

$$\frac{\rho_1 - \rho}{\rho_1 - \rho_2} = \frac{T_1 - T}{T_1 - T_2} \Rightarrow \frac{945 - \rho}{945 - 928} = \frac{120° - 125}{120 - 140} = \frac{-5}{-20}$$

$$(945 - \rho) = \frac{5}{20}(945 - 928)$$

$$\rho_\ell = \left[945 - \frac{5}{20}(945 - 928)\right] = 940.75\ kg/m^3$$

$$\frac{k_1 - k}{k_1 - k_2} = \frac{5}{20} \Rightarrow \frac{0.6850 - k}{0.6850 - 0.6838} = \frac{1}{4}$$

$$(0.6850 - k) = \frac{1}{4}(0.6850 - 0.6838)$$

$$k_\ell = 0.6847\ W/m\text{-}k \text{ at } 125°c$$

For nucleate boiling

$$\frac{Q}{A} = \mu_\ell \times h_{fg} \times \left[\frac{g(\rho_\ell - \rho_v)}{\sigma}\right]^{0.5} \left[\frac{C_\ell \Delta T}{C_{sf}\ h_{fg}\ P_r^n}\right]^3$$

local heat transfer coefficient

$$h_x\, dx\, (T_{sat} - T_s) = k_l\, dx\, \frac{(T_{sat} - T_s)}{\delta(x)}$$

$$h_l = \frac{k_l}{\delta(x)} \rightarrow \text{local heat transfer coefficient}$$

$$h_x = \left[\frac{g \rho_l (\rho_l - \rho_v) k_l^3 h_{fg}}{4 \mu_l (T_{sat} - T_s) x} \right]^{1/4}$$

The average heat transfer coefficient for the entire plate

$$\bar{h}_L = \frac{1}{L} \int_0^l h_x\, dx = \frac{4}{3} h_l$$

$$\bar{h}_L = 0.943 \left[\frac{g \rho_l (\rho_l - \rho_v) k_l^3 h_{fg}}{\mu_l (T_{sat} - T_s) L} \right]^{1/4}$$

Mc Admas proposed a constant based on experimental investigation

$$\bar{h}_L = 1.13 \left[\frac{g \rho_l (\rho_l - \rho_v) k_l^3 h_{fg}}{\mu_l (T_{sat} - T_s) L} \right]^{1/4}$$

The rate of heat transfer in a vertical plate

$$Q = \bar{h}_L A (T_{sat} - T_s)$$

for inclined plates with θ w.r.to ~~horizontal~~ vertical, "g" is replaced with "$g \cos\theta$"

The rate of condensation $m = \dfrac{Q}{h_{fg}} = \dfrac{\bar{h}_L A (T_{sat} - T_s)}{h_{fg}}$

① A heated brass plate at 150 °C is submerged horizontally in

The heat transfer at the wall in the area (elemental)

i.e $dx \times 1$ and considering linear temperature distribution

According to fourier law $q_s = k_l \frac{\Delta T}{\delta} = k_l \times \frac{(T_{sat} - T_s)}{\delta}$

$$dx \times q_s = k_l \frac{(T_{sat} - T_s)}{\delta} dx$$

This is the amount of energy must be transfer from the vapour as it condenses

The heat removed by the wall must equal the incremental mass flow times the latent heat of condensation of the vapour

$$dm \cdot h_{fg} = dx \, q_s$$

$$\frac{\rho_l (\rho_l - \rho_v) g \delta^2 d\delta}{\mu_l} \times h_{fg} = \frac{k_l (T_{sat} - T_s)}{\delta} \times dx$$

$$\delta^3 d\delta = \frac{k_l (T_{sat} - T_s) \times \mu_l}{\rho_l (\rho_l - \rho_v) g \, h_{fg}} dx$$

Boundary conditions $\Rightarrow$ $\delta = 0$ at $x = 0$

$$\int_0^{\delta(x)} \delta^3 d\delta = \frac{k_l \mu_l (T_{sat} - T_s)}{\rho_l (\rho_l - \rho_v) g \cdot h_{fg}} \int_0^x dx$$

$$\frac{\delta^4}{4} = \frac{k_l \mu_l (T_{sat} - T_s)}{\rho_l (\rho_l - \rho_v) g \times h_{fg}} x$$

$$\delta^4 = \frac{4 k_l \mu_l (T_{sat} - T_s) x}{\rho_l (\rho_l - \rho_v) g \, h_{fg}}$$

$$\delta(x) = \left[\frac{4 k_l \mu_l (T_{sat} - T_s) x}{\rho_l (\rho_l - \rho_v) g \, h_{fg}} \right]^{1/4}$$

$$u(y) = \frac{g(\rho_l - \rho_v)\delta^2}{\mu_l}\left[\frac{y}{\delta} - \frac{1}{2}\left(\frac{y}{\delta}\right)^2\right]$$

The condensate mass flow rate through any position in x-direction of the film $\delta(x)$

$$m(x) = \int \rho_l u(y)\, dy\, x = \int_0^{\delta(x)} \rho_l \times \frac{g(\rho_l - \rho_v)\delta^2}{\mu_l}\left[\frac{y}{\delta} - \frac{1}{2}\left(\frac{y}{\delta}\right)^2\right] dy$$

$$m(x) = \frac{\rho_l g(\rho_l - \rho_v) \times \delta^2}{\mu_l} \times \frac{1}{\delta^2}\int_0^{\delta(x)} \left[y\delta - \frac{1}{2}y^2\right] dy$$

$$m(x) = \frac{\rho_l g(\rho_l - \rho_v)}{\mu_l}\left(\delta \frac{y^2}{2} \int_0^{\delta(x)} - \frac{1}{2}\frac{y^3}{3}\right]_0^{\delta(x)}\Bigg)$$

$$m(x) = \frac{1}{2}\frac{\rho_l g(\rho_l - \rho_v)}{\mu_l}\left[\delta \times (\delta^2 - 0) - \frac{1}{3}(\delta^3 - 0)\right]$$

$$m(x) = \frac{1}{2}\frac{\rho_l g(\rho_l - \rho_v)}{\mu_l}\left[\delta^3 - \frac{\delta^3}{3}\right] = \frac{1}{2} \times \frac{\rho_l g(\rho_l - \rho_v)}{\mu_l} \times \frac{2\delta^3}{3}$$

$$m(x) = \frac{\rho_l g(\rho_l - \rho_v)\delta^3}{3\mu_l}$$

mass flow rate for a small element differentiate $m(x)$ w.r.to "δ"

$$dm = \frac{\rho_l g(\rho_l - \rho_v) 3\delta^2 d\delta}{3\mu_l}$$

$$dm = \frac{\rho_l g(\rho_l - \rho_v)\delta^2 d\delta}{\mu_l}$$

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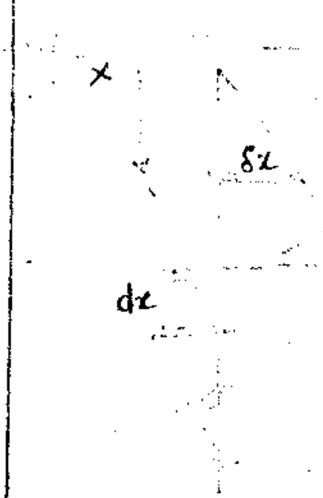
which is less than the saturation temperature (T_{sat}) of vapour

* The condensate flow is laminar
* The fluid properties are constant (like viscosity, specific gravity etc)
* The shear stress at the liquid vapour interface is negligible
* The acceleration of the fluid with in the condensate layer is neglected
* The heat transfer across the condensate layer is by pure conduction and the temperature distribution is linear.



<u>x-momentum equation</u>

$$\rho\left[u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right]=B_x-\frac{\partial p}{\partial x}+\mu_l\left[\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right]$$

$$\frac{\partial^2 u}{\partial y^2}=\frac{1}{\mu_l}\frac{dp}{dx}-\frac{B_x}{\mu_l}$$

$$\boxed{B_x=\rho_l g}$$

$$\frac{dp}{dx}=\rho_v g,\qquad \frac{\partial^2 u}{\partial y^2}=\frac{1}{\mu_l}\times\rho_v g-\frac{1}{\mu_l}\rho_l g$$

$$\Rightarrow \frac{\partial^2 u}{\partial y^2}=\frac{-g}{\mu_l}\left[\rho_l-\rho_v\right]\longrightarrow ①$$

where $\rho_l, \rho_v \rightarrow$ density of liquid and vapour

<u>Boundary conditions</u>

at surface of plate $\Rightarrow u=0$ at $y=0$

and velocity gradient $\frac{\partial u}{\partial y}=0$ at $y=\delta$

Integrate equation ① twice using boundary conditions

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$= (T_{sat} - T_s)$ $T_s < T_{sat} \rightarrow$ condensation

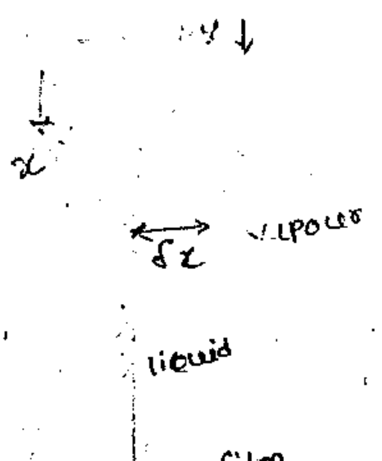
Two modes of condensation

(i) Film wise condensation $\longrightarrow$ condensate wets the surface forming a continuous film with which converts the entire surface

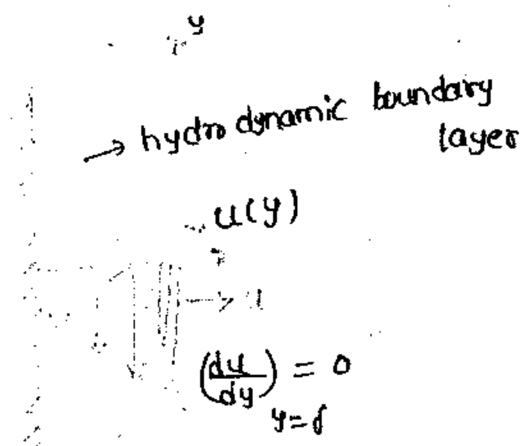
(ii) Drop wise condensation :

vapour condenses into small liquid droplets of varires sizes which full down the surface in a random layer. This is Drop wise condensation.

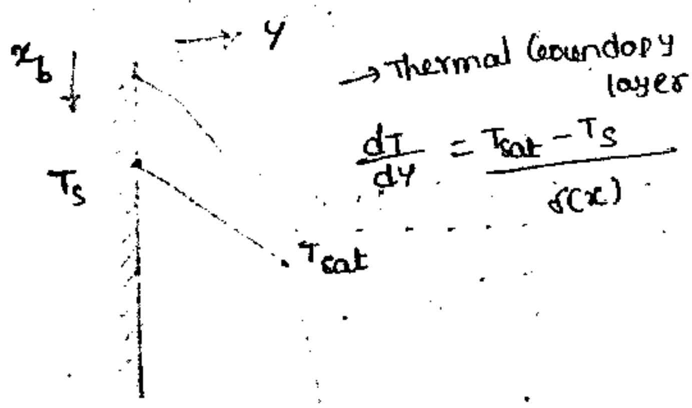
Laminar film wise condensation on a vertical plate (Nusselt theory of condensation)



Growth of film



velocity profile



Temperature profile

Assumptions:

* The plate is maintained at a uniform temperature T_s,

of 10 atm and (b) the pressure is raised to 20 atm at

$\Delta T_e = 10°c$

Sol: given that

ΔT_e = excess temperature = $(T_s - T_{sat}) = 10°c$

heat transfer $h_p = h_a \left[\frac{P}{P_a}\right]^{0.4} = h_a \left(\frac{10}{1}\right]^{0.4}$

$16000 < q_s < 2400 \, W/m^2$, $h_a = 0.56 \, (\Delta T_e)^3$

$h_a = 0.56 \times 10^3$

$h_p = 0.56 \times 10^3 \times 10^{0.4} = 13.97 \; kW/m^2K$

(a) $\Delta T_e = 20°c$ at $P = 10 atm$

$h_p = h_a \times \left[\frac{10}{1}\right]^{0.4} = 0.56 \times 20^3 \, (10)^{0.4}$

$h_p = 111.73 \; kW/m^2k$

heat transfer coefficient is increased to 8 times of the original value

(b) $P = 20 atm$, $\Delta T_e = 10°c$

$h_p = 0.56 \times 10^3 \left[\frac{20}{1}\right]^{0.4} = 18.43 \; kW/m^2k$

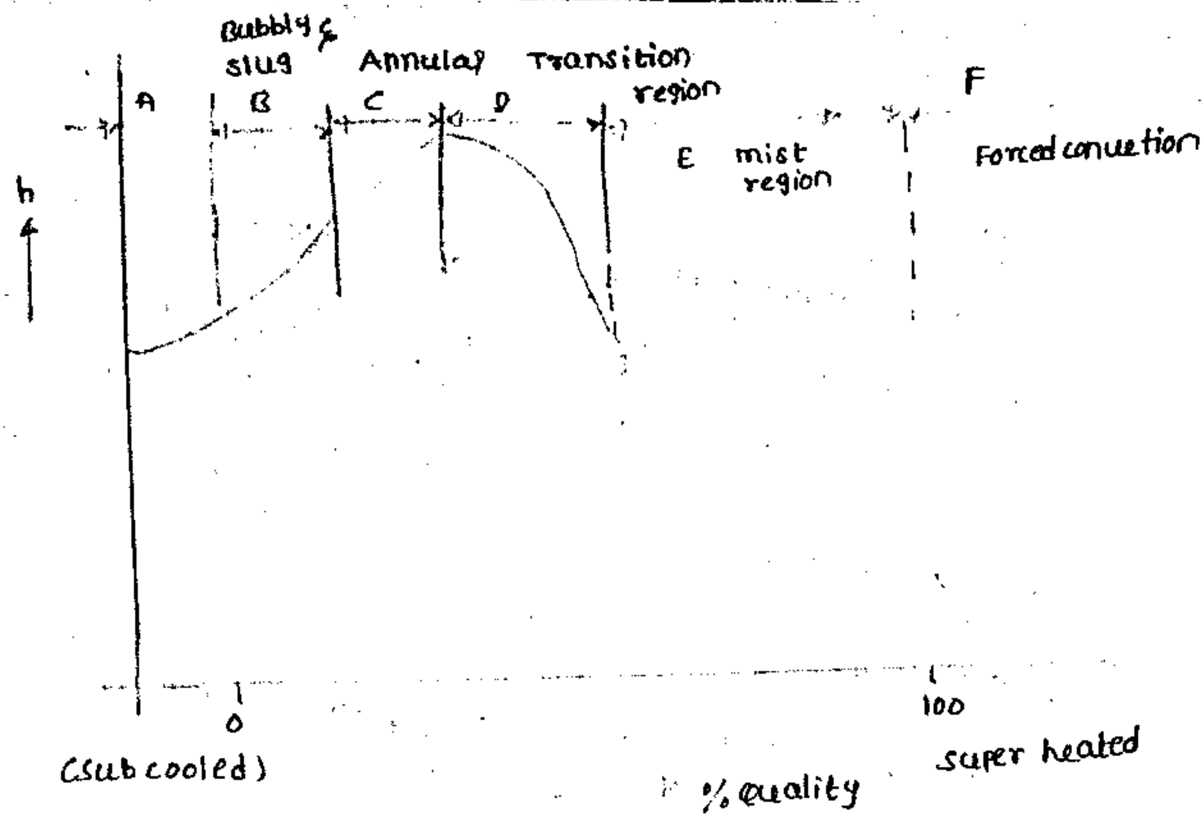
In this heat transfer coefficient is increased by 30% w.r. to the original value

Condensation heat transfer

It is a process which is reverse of boiling

* whenever a saturated vapour comes in content with a surface at a inner temperature condensation occurs

$\Delta T_e \rightarrow$ excess temperature $= (T_s - T_{sat})$ $T_s > T_{sat} \rightarrow$ boiling



for low heat Flux

$$q = 2.253\,(\Delta T_e)^{3.86}\ \text{W/m}^2$$

$$0.2 < P < 0.7\ \text{MN/m}^2$$

for high flux (heat)

$$q = 0.061\,P^{4/3}\,(\Delta T_e)^3\ \text{W/m}^2$$

$$0 < P < 14\ \text{MN/m}^2$$

heat transfer $h_p = h_a \left[\frac{P}{P_a}\right]^{0.4}$

① Calculate the nucleate boiling heat transfer coefficient for water boiling on a horizontal tube whose wall temperature is maintained at 10 °C above the saturation temperature of water. Assume the water to be a pressure of 10atm. Determine the change in value of the heat transfer coefficient when (a) the temperature difference is increased to 20°C at the pressure where $n = 1$ for water

$$q_s = (961 \times 0.293 \times 10^{-6}) \times 2256.9 \times 10^3 \left[9.81 \frac{(961 - 0.598)}{0.0588} \right]^{0.5} \times \left[\frac{4216 \times \Delta T}{0.013 \times 2256.9 \times 10^3 \times 1.74} \right]^3$$

$$\frac{Q}{A} = q_s = \frac{m \times h_{fg}}{A}$$

$m = 30\,kg/hr$, $A = \frac{\pi}{4} d^2$

$d = 30\,cm$

$$q_s = \frac{30 \times (2256.9 \times 10^3)}{3600 \times \frac{\pi}{4} \times (30 \times 10^{-2})^2} = 0.1178 \times 2256.9 \times 10^3$$

$$0.1178 \times 2256.9 \times 10^3 = 2.816 \times 10^{-9} \times 2256.9 \times 10^3 \left[400.288\right] \times 5.632 \times 10^{-4} \times \Delta T^3$$

$$0.1178 = 2.816 \times 10^{-9} \times 400.288 \times 5.632 \times 10^{-4} \times \Delta T^3$$

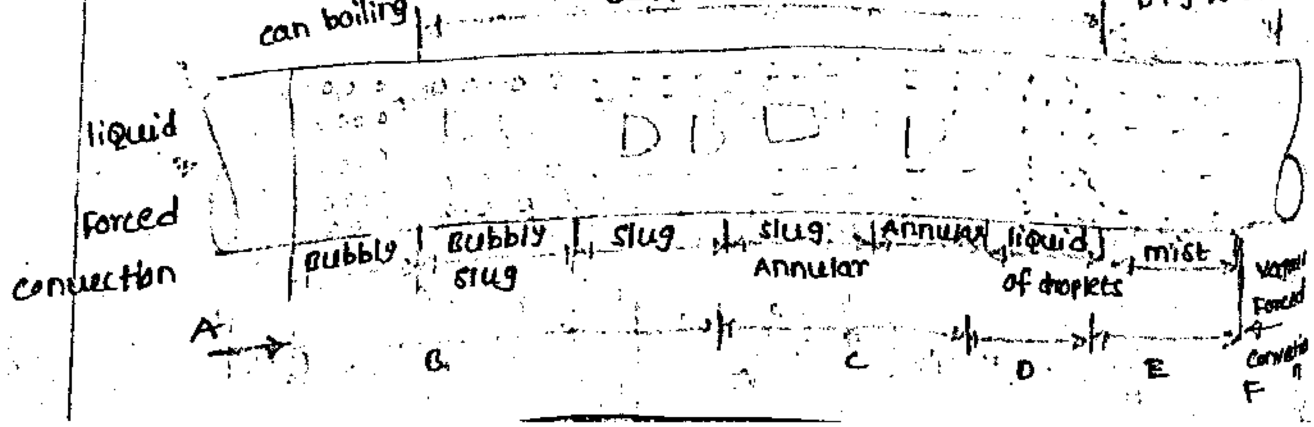
$$0.1178 = 6.3485 \times 10^{-5}\, \Delta T^3 \Rightarrow \Delta T^3 = \frac{0.1178}{6.3485 \times 10^{-5}} = 1855.556$$

$$\Delta T = (1855.556)^{1/3} = 12.288$$

where $\Delta T \rightarrow$ excess temperature $\Rightarrow (T_s - T_{sat}) = 12.288$

$$T_s - 100 = 12.288 \Rightarrow T_s = 100 + 12.288 = 112.288\,°C$$

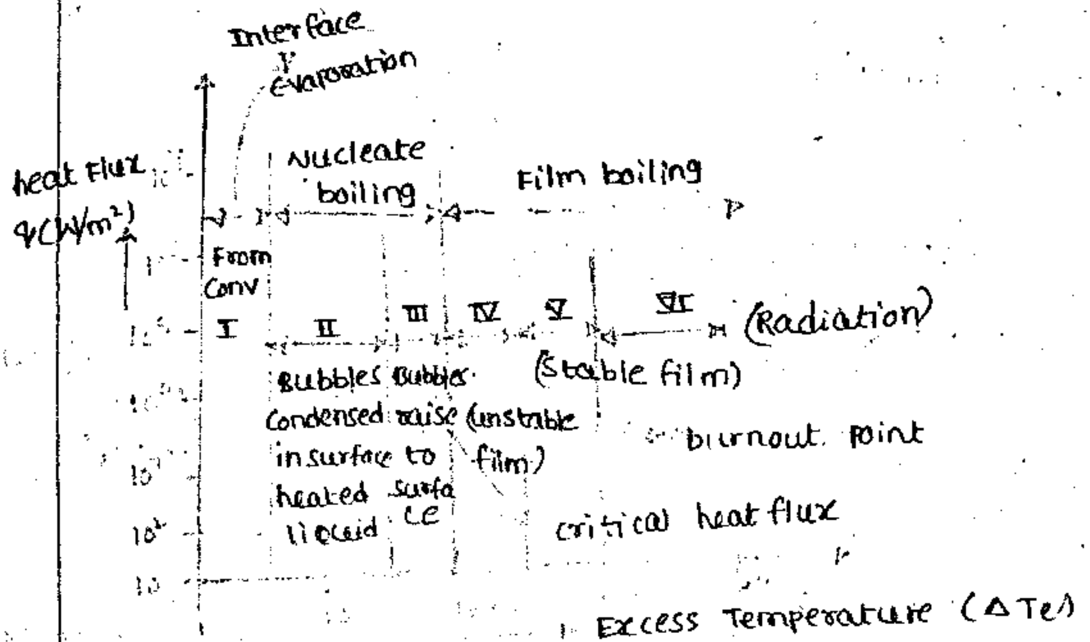
Flow Boiling (or) forced convection boiling



passage.

* There are various distinct reginer of pool boiling in which the heat transfer mechanism differes radically..

Pool boiling → Nucleate boiling, film boiling } stages



① water is boiled at a rate of 30 kg/hr in a copper pan, 30cm in diameter at atmospheric Estimate the temperature of the bottom surface of the pan assuming nucleate boiling conditions

Sol: given that

At 1 atm pressure $T_{sat} = 100°c$ → for water

at 100°c $\rho_l = 961\, kg/m^3$, $\nu_L = 0.293 \times 10^{-6}\, m^2/s$, $P_r = 1.740$

$c_l = 4216\, J/kg\text{-}k$, $\rho_v = 0.598\, kg/m^3$ $(\mu = \rho \times \nu_L)$

at 100°c latent heat $h_{fg} = 2256.9\, kJ/kg$

$$\frac{Q}{A} = \mu_l h_{fg} \left[g \frac{(\rho_l - \rho_v)}{\sigma} \right]^{0.5} \times \left[\frac{c_l \Delta T}{c_{sf} h_{fg} P_r^n} \right]^3$$

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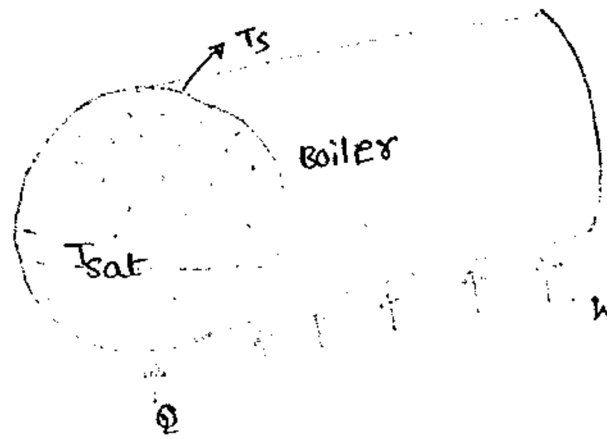
BOILING AND CONDENSATION

Heat transfer phenomenon in Boiling pressure

It is a convection process involving a change in phase from a liquid to vapour. This process may occur when the liquid is in contact with a surface where the surface maintained higher the temperature of saturation liquid



$T_s > T_{sat}$

where $T_s \rightarrow$ surface temperature

$(T_s - T_{sat}) = \Delta T_e \longrightarrow$ excess temperature

According to the newtons law of cooling $Q = hA\Delta T_e$

$$Q = hA(T_s - T_{sat})$$

Types of Boiling process

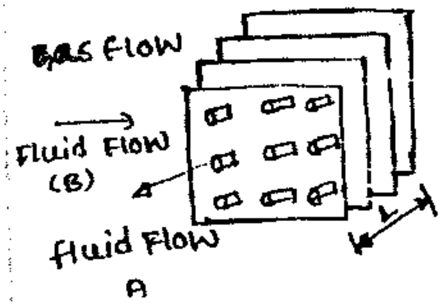
① pool boiling ② Flow boiling

① pool boiling :- Flow of fluid is restricted

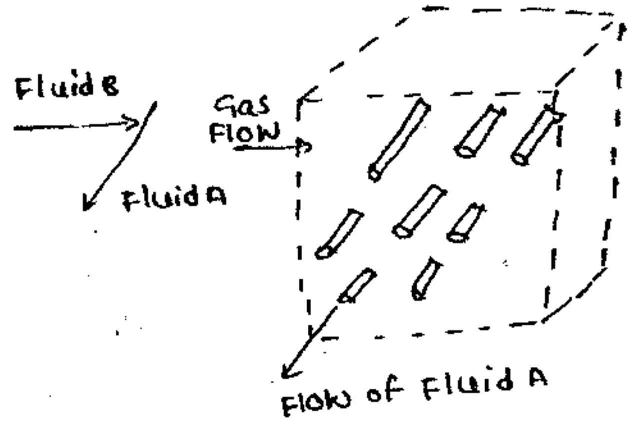
② Flow boiling $\longrightarrow$ Flow of fluid is possible

pool boiling :- If heat is added to a liquid from a submerged solid surface the boiling process is called it as pool boiling.

Flow boiling :- Forced convection boiling occurs in a flow stream and the boiling surface may itself be a portion of the flow

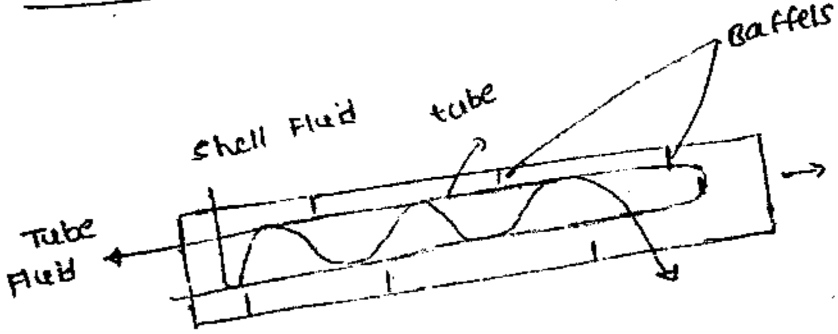


Both Fluids unmixed

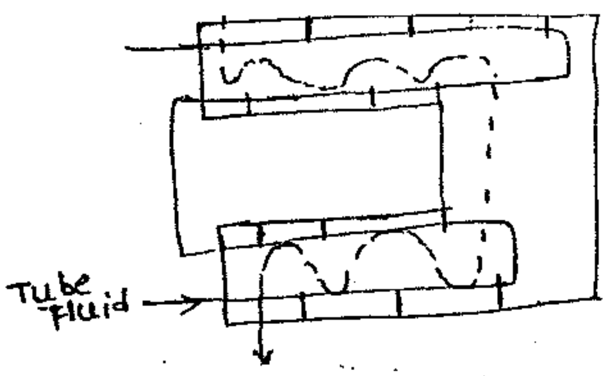


one Fluid unmixed

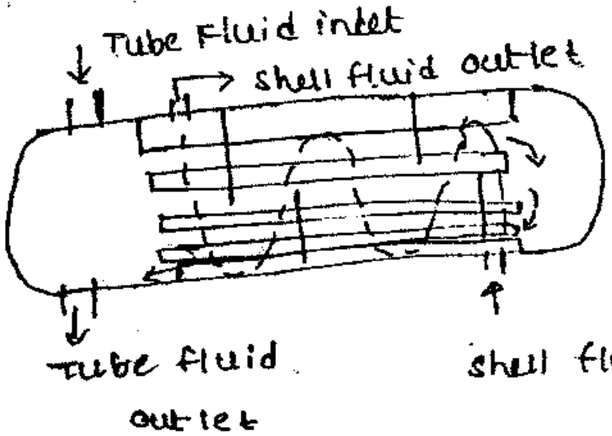
(A) shell and tube type heat exchanger



→ 2 tube pass one shell type



→ 4 tube two shell heat exchanger



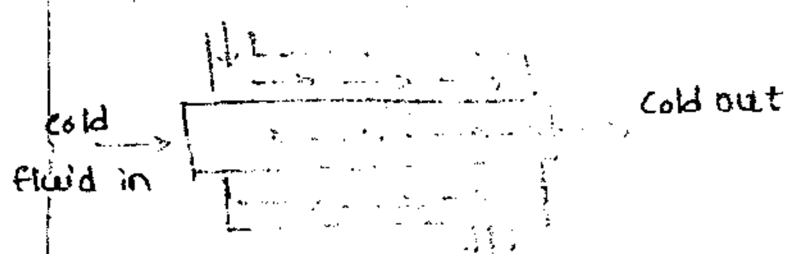
one shell pass, two tube pass type shell tube heat exchanger

Two methods of heat exchanger Analysis

① LMTD method (logirthermic mean temperature difference)

② NTU method (Number of transfer units)

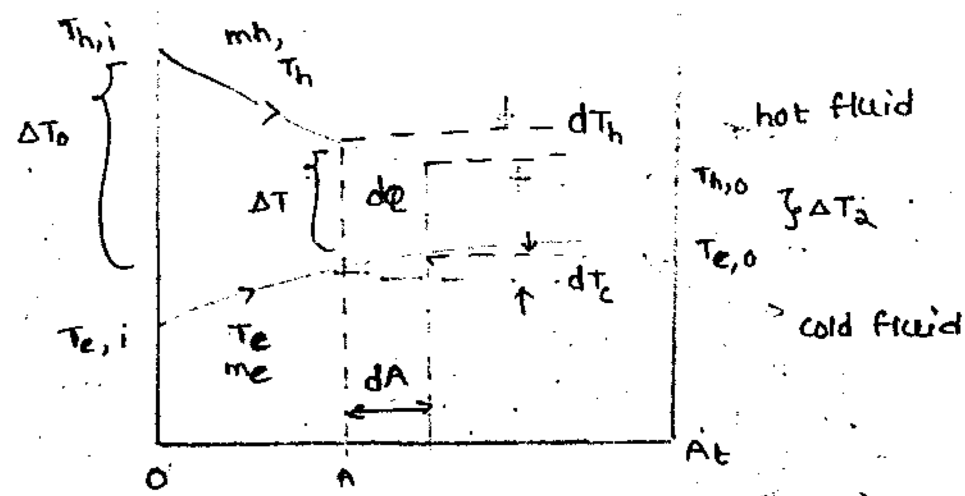
LMTD (logirthermic mean temperature difference) method for heat exchanger analysis



heat transfer rate $Q = AU\Delta T_m$

where $U \rightarrow$ overall heat transfer coefficient

$\Delta T_m \rightarrow$ mean temperature difference



Area from inlet (or) distance from inlet

$A \rightarrow$ heat transfer measure area from inlet

$m_c, m_h \Rightarrow$ mean Flow rate of cold and hot fluids, respectivly

$\Delta T = T_h - T_c$ = local temperature difference b/w cold and hot fluid, $U \rightarrow$ local overall heat transfer between two fluids

The rate of heat transfer dQ from the hot to the cold fluid through an elemental area dA at location A is

$$dQ = U dA \Delta T$$

$dQ \rightarrow$ heat loss by hot fluid = heat gained by cold fluid

$$dQ = -m_h c_{ph} \, dT_h = -m_h C_h \, dT_h \quad \rightarrow \text{hot fluid}$$

$$dQ = m_c c_{pc} \, dT_c = m_c C_c \, dT_c \quad \rightarrow \text{cold fluid}$$

$$\rightarrow \Delta T = T_h - T_c \Rightarrow d(\Delta T) = dT_h - dT_c$$

$$dT_h = \frac{-dQ}{m_h C_h}, \quad dT_c = \frac{dQ}{m_c C_c} \Rightarrow d(\Delta T) = \frac{-dQ}{m_h C_h} - \frac{dQ}{m_c C_c}$$

$$d(\Delta T) = -dQ\left[\frac{1}{m_h C_h} + \frac{1}{m_c C_c}\right] = -dQ \, B$$

where $B = \left[\frac{1}{m_h C_h} + \frac{1}{m_c C_c}\right]$

$$d(\Delta T) = -B \, dQ$$

To eliminate "dQ"

$$\frac{d(\Delta T)}{\Delta T} = \frac{-B \, dQ}{\frac{dQ}{U dA}} = -BU dA$$

$$\int_{\Delta T_0}^{\Delta T_L} \frac{d(\Delta T)}{\Delta T} = \int_0^{A_t} -BU dA$$

$$\int_{\Delta T_0}^{\Delta T_L} \frac{d(\Delta T)}{\Delta T} = -B \times A_t \int_0^{A_t} \frac{U dA}{A_t}$$

average overall heat transfer coefficient $U_m = \frac{1}{A_t} \int_0^{A_t} U dA$

$$\int_{\Delta T_0}^{\Delta T_L} \frac{d(\Delta T)}{\Delta T} = -B U_m A_t \qquad U_m = \frac{1}{A_t}\int_0^{A_t} U\,dA$$

$$\boxed{d\varphi = U_m A \times \Delta T}$$

$$\ln(\Delta T)\Big]_{\Delta T_0}^{\Delta T_L} = -B U_m A_t$$

$$\ln\left[\frac{\Delta T_L}{\Delta T_0}\right] = -B U_m A_t \Rightarrow \ln\left[\frac{\Delta T_0}{\Delta T_L}\right] = B U_m A_t$$

$$\downarrow$$

$$B = \frac{\ln(\Delta T_0/\Delta T_L)}{U_m A_t}$$

To calculate total heat transfer rate

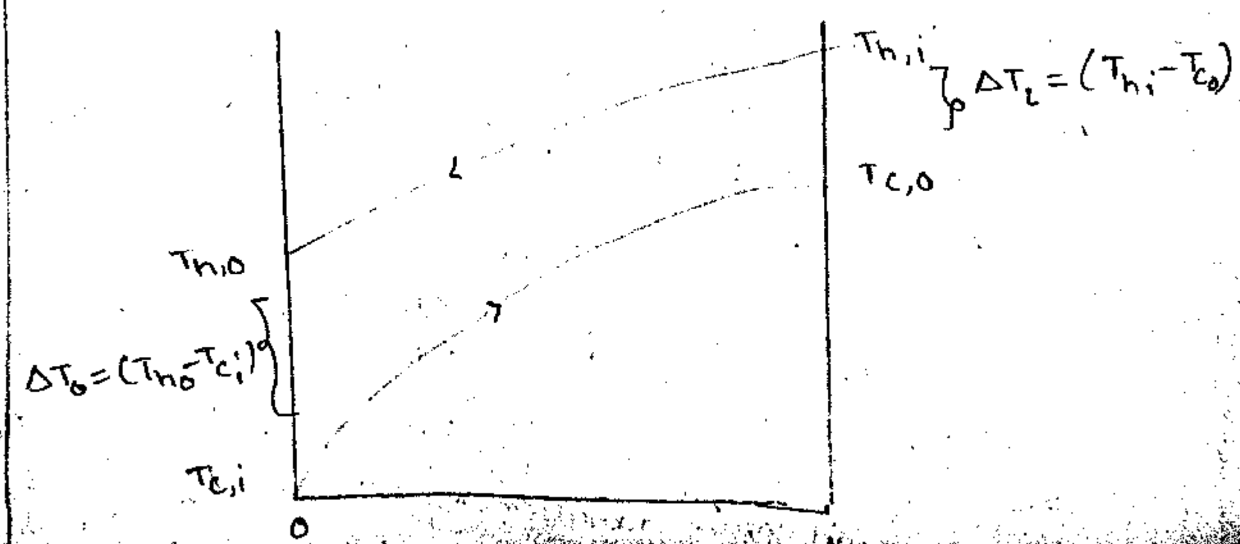
$$d(\Delta T) = -B\,\varphi$$

$$\int_{\Delta T_0}^{\Delta T_L} d(\Delta T) = -B\int_0^{\varphi} d\varphi$$

$$(\Delta T_L - \Delta T_0) = -B\varphi \Rightarrow B\varphi = (\Delta T_0 - \Delta T_L)$$

$$\varphi = \frac{(\Delta T_0 - \Delta T_L)}{\frac{\ln(\Delta T_0/\Delta T_L)}{U_m A_t}} \Rightarrow \varphi = A_t U_m \frac{(\Delta T_0 - \Delta T_L)}{\ln(\Delta T_0/\Delta T_L)}$$

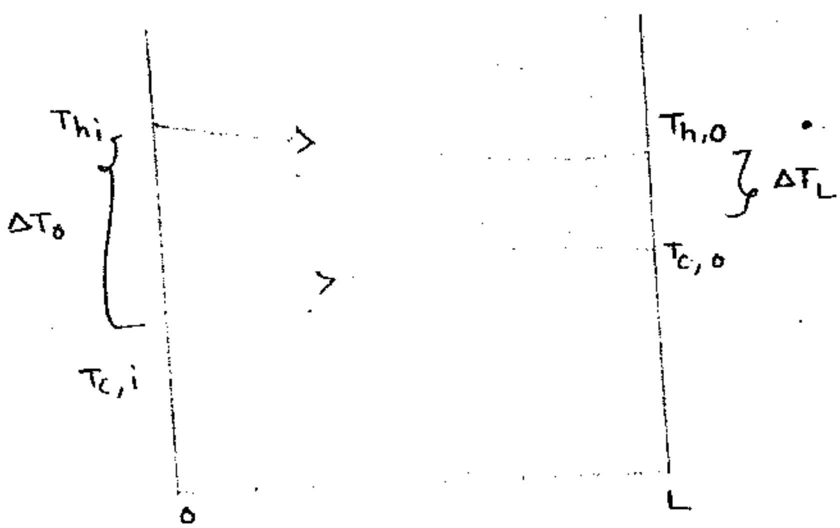
$$\frac{(\Delta T_0 - \Delta T_L)}{\ln(\Delta T_0/\Delta T_L)} = \Delta T_{ln}$$ → logirthermic mean temperature difference



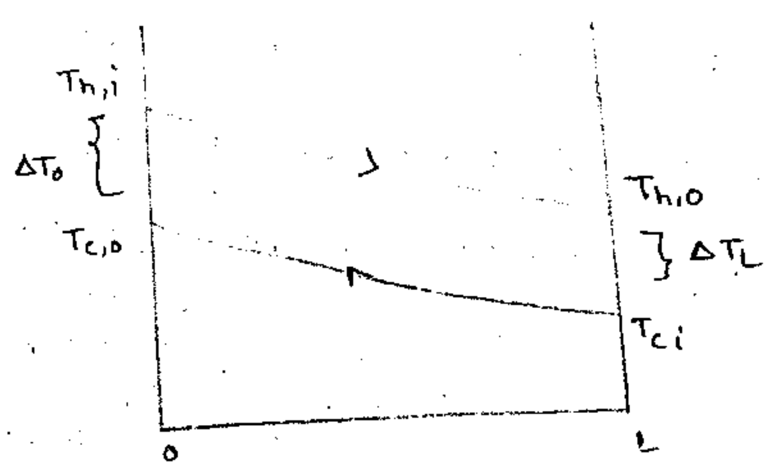
① In a single pass shell and tube heat exchanger the inlet and outlet temperature for the hot fluid are $T_{hi} = 260°C$ and $T_{ho} = 140°C$, for the cold fluid they are $T_{c,i} = 70°C$ and $T_{c,o} = 125°C$ calculate the logirthermic mean temperature difference for (a) counter flow and (b) parallel flow arrangements

Sol:- given that

parallel flow



Counter flow



$$\Delta T_{ln} = \frac{\Delta T_0 - \Delta T_L}{ln(\Delta T_0/\Delta T_L)}$$

parallel flow

$\Delta T_0 = T_{hi} - T_{ci} = 260 - 70 = 190°C$

$\Delta T_c = \Delta T_{ho} - \Delta T_{co} = 140 - 125 = 15°C$

$$\Delta T_{ln} = \frac{190-15}{ln\left(190/15\right)} = 60.9$$

Counter Flow

$$\Delta T_0 = T_{hi} - T_{co} = 260 - 125 = 135^0$$

$$\Delta T_L = T_{ho} - T_{ci} = 140 - 70 = 70°c$$

$$\Delta T_{ln} = \frac{135-70}{ln\left[\frac{135}{70}\right]} = 99$$

② A counter flow shell and tube heat exchanger is used to heat water at a rate of $m = 0.8 kg/s$ from $T_i = 35°c$ to $T_o = 80°c$ with hot oil entering at 120°c and leaving at 85°c the overall heat transfer coefficient is $U = 125\ W/m^2°c$ calculate the heat transfer area required

Sol:- given that $m_c = 0.8 kg/s$, $T_{c,i} = 30°c$, $T_{c,o} = 80°c$

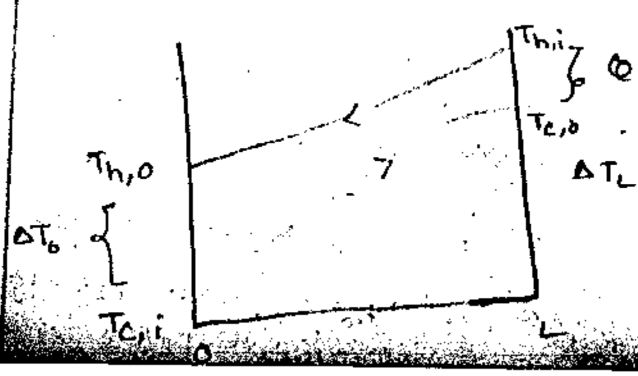
$T_{h,i} = 120°c$, $T_{h,o} = 85°c$, $U = 125\ W/m^2°c$

specific heat of water $c_p = 4180\ J/kg°c = C_c$

total heat transfer rate $Q = m_c C_c (T_{c,o} - T_{c,i})$

$$= 0.8 \times 4180\ (80-30)$$

$$Q = 167,200\ W$$



$$\Delta T_{ln} = \frac{\Delta T_o - \Delta T_L}{ln\left[\Delta T_o / \Delta T_L\right]}$$

$$\Delta T_o = T_{h_o} - T_{c_i} = 85 - 30 = 55°C$$

$$\Delta T_L = T_{h_i} - T_{c_o} = 120 - 80 = 40°C$$

$$\Delta T_{ln} = \frac{55-40}{ln\left[55/40\right]} = 47.1°C$$

W.K.T $Q = UA_t \Delta T_{ln}$

$$A_t = \frac{Q}{U \Delta T_{ln}} = \frac{167200}{125 \times 47.1} = 28.4\ m^2$$

(3) An oil cooler for a large diesel engine is to cool engine oil from 60 to 45°C, using sea water at an inlet temperature of 20°C with temperature rise of 15°C the design heat load is $Q = 140$ KW and the mean overall heat transfer coefficient based on the outer surface area of the tubes is $70\ W/m^2\,°C$. Calculate the heat transfer surface area for single pass (a) Counter Flow (b) parallel Flow arrangements

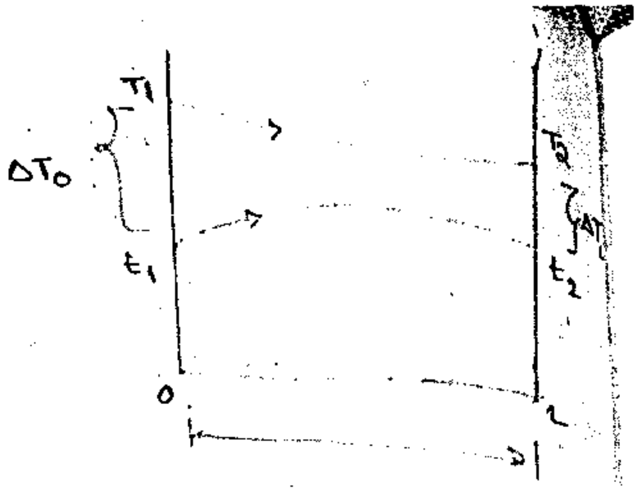
Sol:- given that

let $T_1 \rightarrow$ hot inlet temperature $= 60°C$, $T_2 \rightarrow$ hot exit temperature $= 45°C$, $t_1 \rightarrow$ cold inlet temperature $= 20°C$, $t_2 \rightarrow$ cold exit temperature $= 35°C$, $Q = 140\,KW$, $U_m = 70\,W/m^2\,°C$

parallel Flow

$$\Delta T_{ln} = \frac{\Delta T_o - \Delta T_L}{ln\left(\frac{\Delta T_o}{\Delta T_L}\right)}$$

$$\Delta T_{ln} = \frac{(T_1 - t_1) - (T_2 - t_2)}{\ln\left[\frac{(T_1 - t_1)}{(T_2 - t_1)}\right]}$$



$$\Delta T_{ln} = \frac{(60-20)-(45-35)}{\ln\left[\frac{40}{10}\right]}$$

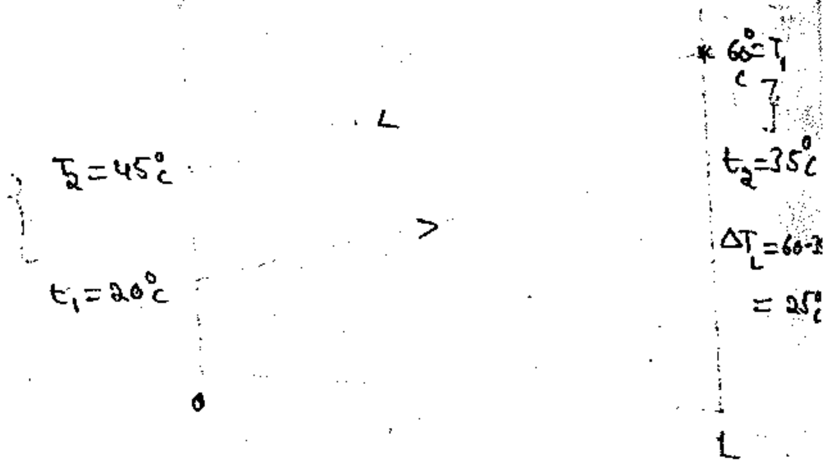
$$\Delta T_{ln} = \frac{40-10}{\ln(4)} = \frac{30}{\ln(4)} = 21.6\ °C$$

W.K.T $Q = U_m A \Delta T_{ln} \Rightarrow 140\times10^3 = 70 \times A \times 21.6$

$$A = \frac{140\times10^3}{70\times21.64} = 92.42\,m^2$$

(ii). Counter Flow

$\Delta T_0 = 45-20$
$= 25°C$



$$\Delta T_{lm} = \frac{(\Delta T_0 - \Delta T_L)}{\ln\left(\Delta T_0 / \Delta T_L\right)}$$

$\therefore \Delta T_0 = \Delta T_L = 25°C$

$\Delta T_{lm} = 25°C$

W.K.T $Q = U_m A \Delta T_{lm}$

$$140\times10^3 = 70\times A\times 25 \Rightarrow A = \frac{140\times10^3}{70\times25} = 80\,m^2$$

(4) Engine oil is to be cooled from 80 to 50°C by using a single pass counter flow, concentric tube heat exchanger

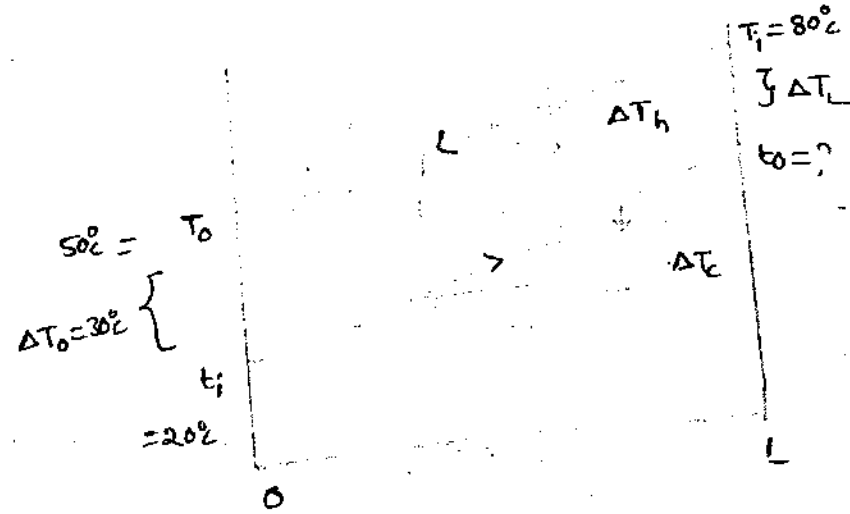
with cooling water available at 20°c water inside a tube with ID of $D_i = 2.5cm$ at a rate of $m_w = 0.08 kg/s$ and flows through the annulus at a rate of $m_o = 0.16 kg/s$. The heat transfer coefficients for the water side and oil side are respectively $h_w = 1000 W/m^2$, $h_o = 80 W/m^2 °c$, the fouling factors are $F_w = 0.00018 \frac{m^2}{cW}$; $F_o = 0.00018 m^2/wc$ and the tube wall resistance is negligible. calculate the tube length require $C_w = C_c = 4180 J/kg°c$ $C_o = C_h = 2090 J/kg°c$

sol: given that

$C_o = C_h = 2090 J/kg°c$

$C_w = C_c = 4180 J/kg°c$

$$\frac{C_{min}}{C_{max}} = \frac{C_h}{C_c} = \frac{2090}{4180} = 0.5$$



heat loss

$Q = m C_p \Delta T$

W.K.T $Q = m_{oil} C_{oil} (T_{h,i} - T_{h,o})$

$(T_{hi} - T_{ho}) = \Delta T_h$, $(T_{co} - T_{ci}) = \Delta T_c$

heat gained

$Q = m_w C_w (T_{co} + T_{ci})$

$Q_{loss} = Q_{gain}$

$$m_o c_o (T_{h_i} - T_{h_o}) = m_w c_w (T_{c_o} - T_{c_i})$$

$$0.16 \times 2090 \; (80-50) = 0.08 \times 4180 \; (T_{c,o} - 20^\circ)$$

$$T_{c,o} = 50^\circ C$$

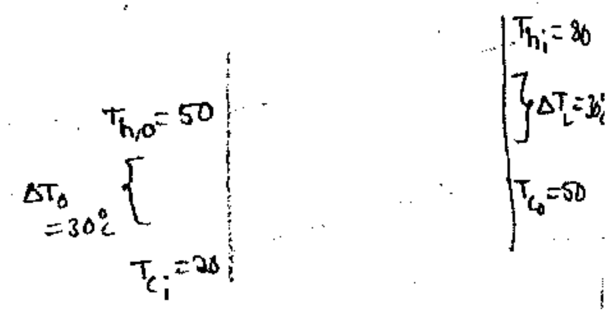
W.K.T $\quad Q = 0.08 \times 4180 \; (50-20)$

$$\boxed{Q = 10032 \text{ W}}$$

W.K.T $\quad$ heat gained $\quad Q = U_m A \, \Delta T_{lm}$

$$\Delta T_0 = \Delta T_L = 30^\circ C \quad \Rightarrow \quad \Delta T_{lm} = 30^\circ$$

W.K.T
overall heat transfer coefficient

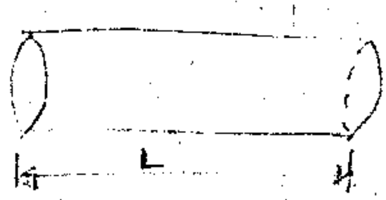


$$U = \frac{1}{\frac{1}{h_w} + F_w + F_{oil} + \frac{1}{h_{oil}}}$$

$$= \frac{1}{\left(\frac{1}{1000}\right) + 0.00018 + 0.00018 + \frac{1}{80}} = 72.2 \text{ W/m}^2 {}^\circ C$$

$$Q = U_m A \, \Delta T_{lm} \quad \Rightarrow \quad 10032 = 72.2 \times A \times 30$$

$$A = \frac{10032}{72.2 \times 30} = 4.6315 \text{ m}^2$$



(A) Area $= \pi D L$

$$L = \frac{A}{\pi D_i} = \frac{4.6315}{\pi \times (2.5 \times 10^{-2})}$$

$$L = 59.01 \simeq 60 \text{ m}$$

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(5) A shell and tube steam condensor is to be constructed of 2.5cm outer Diameter, 2.2cm Inner diameter single pass horizontal tubes with steam condensing at $T_s = 54°c$ outside the tubes. The cooling water enters each tube at $T_i = 18°c$, with flow rate of $m = 0.7 kg/s$ per tube and leaves at $T_0 = 36°c$. The heat transfer coefficient for the condensation of steam is $h_s = 8000 W/m^2 °C$. calculate the tube length. calculate the condensation rate per tube

NTU method for heat exchanger Analysis

Effectiveness $\epsilon = \dfrac{\text{Actual heat transfer rate}}{\text{maximum possible heat transfer rate}} = \dfrac{Q}{Q_{max}}$

The maximum possible heat transfer rate obtained with a counter Flow heat exchanger if the temperature change of a fluid having the minimum value of mc_p equal to the difference in inlet temperature of hot and cold fluid

$$Q = mc_p \Delta T$$

$$Q_{max} = (mc_p)_{min} (T_{hi} - T_{ci})$$

Actual heat transfer $Q = \epsilon Q_{max}$

$$Q = \epsilon (mc_{p\,min})(T_{hi} - T_{ci})$$

$c_{p\,min} \rightarrow$ taken based on the smallest values of C_h (or) C_c

$C_h \rightarrow$ specific heat of hot fluid

$C_c \rightarrow$ specific heat of cold fluid

$$\epsilon = \frac{Q}{(mc_p)_{min} (T_{h,i} - T_{c,i})}$$

$$Q = m_h c_h (T_{hi} - T_{h,o}) = m_c c_c (T_{c,o} - T_{ci}) \Rightarrow \text{Actual}$$

$$E = \frac{m_h c_h (T_{hi} - T_{h,o})}{(mc_p)_{min} (T_{hi} - T_{ci})} \Rightarrow E = \frac{c_h (T_{hi} - T_{ho})}{c_{min} (T_{hi} - T_{ci})}$$

(or)

$$E = \frac{c_c (T_{co} - T_{ci})}{c_{min} (T_{hi} - T_{ci})}$$

$c_c = m_c c_c$

$c_h = m_h c_h$

$c_{min} = (mc_p)_{min}$

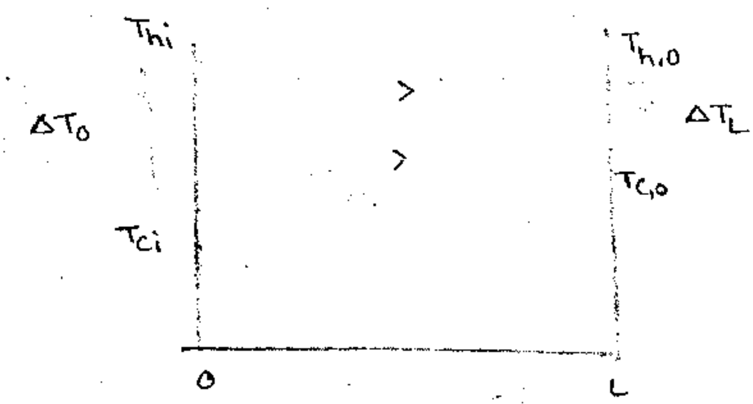
w.k.T $\ln\left(\frac{\Delta T_0}{\Delta T_L}\right) = B U_m A_t$

$$B = \frac{1}{m_h c_h} + \frac{1}{m_c c_c}$$

$\Delta T_0 = T_{hi} - T_{ci}$

$\Delta T_L = T_{ho} - T_{co}$

For parallel Flow



$$\ln\left(\frac{\Delta T_0}{\Delta T_L}\right) = B U_m A_t \Rightarrow \frac{\Delta T_0}{\Delta T_L} = e^{(B U_m A_t)}$$

$$\frac{T_{hi} - T_{ci}}{T_{ho} - T_{co}} = e^{B U_m A t}$$

$$\frac{(T_{ho} - T_{co})}{(T_{hi} - T_{ci})} = e^{-B U_m A t}$$

w.k.T $Q = m_h c_h (T_{hi} - T_{ho}) = m_c c_c (T_{co} - T_{ci})$

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$$Q = C_h (T_{hi} - T_{ho}) = C_c (T_{co} - T_{ci})$$

$$(T_{hi} - T_{ho}) = \frac{C_c}{C_h}(T_{co} - T_{ci})$$

$$T_{ho} = T_{hi} - \frac{C_c}{C_h}(T_{co} - T_{ci})$$

$$\frac{T_{ho} - T_{co}}{T_{hi} - T_{ci}} = e^{-\frac{BUA}{m}t} \Rightarrow \frac{\left[T_{hi} - \frac{C_c}{C_h}(T_{co} - T_{ci})\right] - T_{co}}{(T_{hi} - T_{ci})} = e^{-\frac{BUA}{m}t}$$

$$\frac{T_{hi} - \frac{C_c}{C_h}T_{co} + \frac{C_c}{C_h}T_{ci} - T_{co}}{(T_{hi} - T_{ci})} = e^{-\frac{BUA}{m}t}$$

$$\frac{\left(T_{hi} + \frac{C_c}{C_h}T_{ci}\right) - T_{co}\left[\frac{C_c}{C_h} + 1\right]}{(T_{hi} - T_{ci})} = e^{-\frac{BUA}{m}t}$$

$$1 - \frac{(T_{co} - T_{ci})}{(T_{hi} - T_{ci})}\left[1 + \frac{C_c}{C_h}\right] = e^{-\frac{BUA}{m}t}$$

$$\frac{(T_{co} - T_{ci})}{(T_{hi} - T_{ci})}\left[1 + \frac{C_c}{C_h}\right] = 1 + e^{-\frac{BUA}{m}t}$$

$$\frac{(T_{co} - T_{ci})}{(T_{hi} - T_{ci})} = \frac{1 + e^{-BUmA}}{\left(1 + \frac{C_c}{C_h}\right)}$$

$$\varepsilon = \frac{C_c (T_{co} - T_{ci})}{C_{min}(T_{hi} - T_{ci})} \quad \text{or} \quad \frac{C_h (T_{hi} - T_{ho})}{C_{min}(T_{hi} - T_{ci})}$$

$$E = \frac{\left[1 - e^{-BU_mA}\right]}{\left[\frac{C_{min}}{C_c} + \frac{C_{min}}{C_h}\right]} \qquad B = \frac{1}{C_c} + \frac{1}{C_h}$$

NTU Relations: It is a dimensionless parameter called number of transfer units

$$BU_mA = N\left[\frac{C_{min}}{C_c} + \frac{C_{min}}{C_h}\right]$$

$$N \cong NTU = \frac{AU_m}{C_{min}} \qquad U_mA = NC_{min}$$

$$E = \frac{1 - \exp\left[-N\left(\frac{C_{min}}{C_c} + \frac{C_{min}}{C_h}\right)\right]}{\left[\frac{C_{min}}{C_c} + \frac{C_{min}}{C_h}\right]}$$

① Water enters a counter flow double pipe heat exchanger at 15°C flowing at the rate of 1300 kg/hr, It is heated by oil $C_p = 2000$ J/kg-k flowing at a rate of 550 kg/hr from the inlet temperature of 94°C. for an area of 1m² and an overall heat transfer coefficient of 1075 W/m²k. Determine the total heat transfer and the outlet temperatures of water and oil $(C_{p_1})_w = 4186$ J/kg k

Sol:- given that $T_{h_i} = T_1 = 94°C$ $\quad t_1 = 15°C = T_{c_i}$

$T_{h,o} = T_2 = ?$ $\quad t_2 = ? = T_{co}$

$m_c = 1300\ kg/hr = \frac{1300\ kg}{3600\ sec}$, $C_{P_W} = 4186\ J/kg\text{-}k$

for water

$C_c = m_c \times (C_{P_c}) \Rightarrow C_c = m_c \times C_c = \frac{1300}{3600} \times 4186 = 1511.6\ W/s$

$m_h = 550\ kg/hr = \frac{550}{3600}$ $\quad C_p = 2000\ J/kg\text{-}k = C_h$

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For oil

$$C_h = m_h c_h = \frac{550}{3600} \times 2000 = 305.55 \; W/k$$

$$C_c = 1511.61 \frac{W}{k}, \quad C_h = 305.55 \; W/k$$

$$C_h < C \Rightarrow C_{min} = C_h, \quad C_{max} = C_c$$

$$C = \frac{C_{min}}{C_{max}} = \frac{305.55}{1511.61} = 0.2$$

Calculating NTU

$$NTU = \frac{U_m A}{C_{min}} = \frac{1075 \times 1}{305.55} = 3.52$$

$$N = NTU = 3.52$$

For counter flow effectiveness

$$\varepsilon = \frac{1 - \exp[-N(1-C)]}{1 - C\exp[-N(1-C)]}$$

$$\varepsilon = \frac{1 - e^{[-3.52(1-0.2)]}}{1 - 0.2 \times e^{[-3.52(1-0.2)]}} = \frac{1 - 0.059}{1 - 0.2 \times 0.059} = 0.953$$

$$\varepsilon = 0.953$$

$$Q_{max} = C_{min}(T_1 - t_1) = 305.55(94 - 15) = 24138.5 W$$

$$\varepsilon = \frac{Q_{Act}}{Q_{max}} \Rightarrow Q_{Act} = \varepsilon Q_{max} = 0.953 \times 24138.5 = 22690.2 W$$

$$Q_{Act} = 22690.2 W$$

heat gained by water

$$Q = m_c c_c (T_{co} - T_{ci}) = C_c [t_2 - t_1]$$

$$Q = C_c [T_2 - t_1] \Rightarrow 22690.2 = 1511.6 [t_2 - 15]$$

$$t_2 = T_{c,o} = 30.01 °C$$

$$Q = m_h c_h (T_{h,i} - T_{h,o}) = C_h (T_1 - T_2)$$

$$22690.2 = 305.55 [94 - T_2]$$

$$T_2 = 94 - 74.26 = 19.74°c$$

② In an industry 0.6 kg/sec of oil ($C_p = 2.5$ kJ/kg-k) to be cooled in a counter flow heat exchanger from 110°c to 35°c by the use of water entering at 20°c. The overall heat transfer coefficient is 1500 W/m²-k. presuming the exit water temperature should not exceeds 80°c using NTU method calculate (i) water flow rate (ii) surface are required (iii) The effectiveness of heat exchanger

Sol:- given that

$m_h = 0.6$ kg/s $c_h = 2.5 \times 10^3$ J/kg-k } oil

$T_1 = T_{hi} = 110°c$ $T_2 = T_{ho} = 35°c$

$m_c = ?$ $c_c = ?$ } water

$t_1 = T_{ci} = 20°c$, $t_2 = T_{co} = 80°c$

$U = 1500$ W/m²k

① water flow rate

C_p of water at 20°c (saturated condition) $c_c = 4178$ J/kg-k

$$Q = m_h C_h \times (T_{hi} - T_{ho}) \Rightarrow Q = 0.6 \times 2.5 \times 10^3 (110 - 35)$$

$$Q = 112500 \text{ W} = m_c C_c (T_{co} - T_{ci})$$

$$112500 = m_c \times 4178 (80 - 20) \Rightarrow m_c = 0.449 \text{ kg/s}$$

$$C_h = m_h \times C_h = 0.6 \times 2.5 \times 10^3 = 1500 \text{ W/k}$$

$$C_c = m_c \times C_c = 0.449 \times 4178 = 1875.92 \text{ W/k}$$

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$C_h < C_c \Rightarrow C_{min} = C_h = 1500 \ W/K$

$$N = \frac{UA}{C_{min}} = \frac{1500 \times A}{1500} \Rightarrow N = A$$

(ii) effectiveness of heat exchanger

$$\frac{C_{min}}{C_{max}} = C = \frac{1500}{1875.92} = 0.78$$

$$\text{Effectiveness } \varepsilon = \frac{Q_A}{Q_{max}} = \frac{112500}{Q_{max}}$$

$$Q_{max} = C_{min} \times (T_{h_i} - T_{c_i}) = 1500 \times (110 - 80) = 45000$$

$$\varepsilon = \frac{m_h C_h}{C_{min}} \left[\frac{T_1 - T_2}{T_1 - t_1}\right] = \frac{0.6 \times 2.5 \times 10^3}{1500} \left[\frac{110 - 35}{110 - 20}\right] = 0.83$$

(iii) surface area required

$$\varepsilon = \frac{1 - \exp[-N(1-C)]}{1 - C\exp[-N(1-C)]} , \quad N = \frac{1}{(C-1)} \ln\left[\frac{(\varepsilon - 1)}{(C\varepsilon - 1)}\right]$$

$$N = \frac{1}{(0.78-1)} \times \ln\left[\frac{0.83-1}{(0.78 \times 0.83 - 1)}\right] = \frac{1}{(0.78-1)} \times (-0.7205)$$

$N = 2.315 \Rightarrow N = A \Rightarrow A = 3.315 \ m^2$

③ Refrigeration is designed to cool 250 kg/hr of hot liquids of heat 3350 J/kg-K at 120°c using a parallel flow arrangement 1000 kg/hr of cooling water is available for cooling purpose at a temperature of 10°c if the overall heat transfer coefficiement is 1160 W/m²K and the surface area of the heat exchanger is $0.25 m^2$. calculate the outlet temperature of the cooled liquid and water and also effectiveness of the heat exchanger

sol:- given that

$m_h = 250$ kg/hr $\quad C_{p_h} = 3250 J/kgK$, $C_h = m_h C_h$

$= 0.06945 kg/s$

$T_{h_i} = 120°c$, $m_c = 1000 kg/hr = 0.2788 kg/s$, $C_c = 4178 J/kg\text{-}k$

$U = 1160 W/m^2 K$, $A = 0.25 m^2$

W.K.T $\quad N = \dfrac{UA}{C_{min}}$ $\qquad C_h = m_h C_h = 0.06945 \times 3250$

$= 225.69 W/k = C_{min}$

$C_c = m_c C_c = 0.2778 \times 4178 = 1160.648 W/k = C_{max}$

$C_{min} = 225.69 W/k$

here $N = \dfrac{UA}{C_{min}} = \dfrac{1160 \times 0.25}{225.69} = 1.28$

effectiveness

$$\varepsilon = \frac{1 - exp[-N(1+c)]}{1+c} = \frac{1 - exp[-1.28(1+0.194)]}{1+0.194}$$

$= 0.6558$

$$c = \frac{C_{min}}{C_{max}} = \frac{225.69}{1160.648} = 0.194$$

$Q_{max} = C_{min}(T_1 - t_1)$

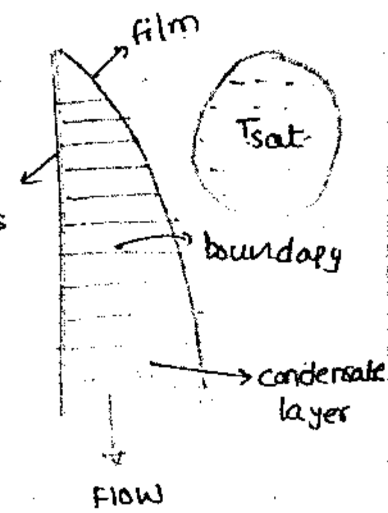
$= 225.69 (120° - 10°) = 24825.9 W$

$$\varepsilon = \frac{Q_{act}}{Q_{max}} \Rightarrow Q_{act} = \varepsilon \times Q_{max} = 0.6558 \times 24825.9 = 16280.82\,W$$

$$Q = m_c c_c \times (T_{co} - T_{ci}) = C_c (t_2 - t_1)$$

Nusselt theory of Condensation

Assumptions



① The plate is maintained at a uniform temperature T_s (or) T_w that is less than the saturation temperature T_s T_{sat} of the vapour

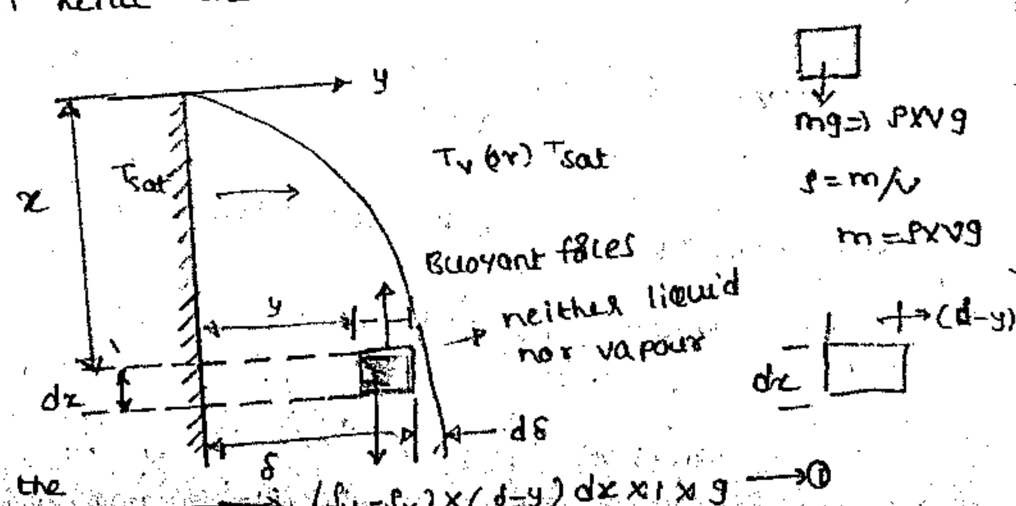
② The vapour is stationary (low velocity) so it exerts no drag on the motion of the condensation

③ The downward flow of condensate under the action of gravity is laminar

④ The flow velocity is associated with condensate film is low as acceleration of the flow is negligible

⑤ Fluid properties are constant

⑥ heat transfer accross the condensate layer is by pure conduction hence the liquid temperature distribution is linear



force under the action of gravity $\longrightarrow (\rho_L - \rho_V) \times (\delta - y)\, dx \times 1 \times g \longrightarrow ①$

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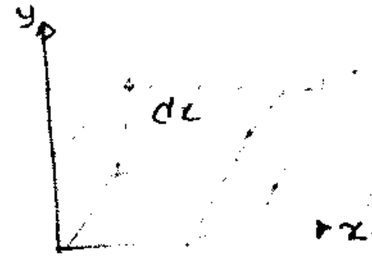
According to the newtons law of viscosity

$$\tau = \mu_l \times \frac{\partial u}{\partial y} \qquad \tau \times A = \text{shear force}$$



$$\mu_l \times \frac{\partial u}{\partial y} \times dx \times 1 = \text{shear force} \longrightarrow (2)$$

Buoyant force = gravity force

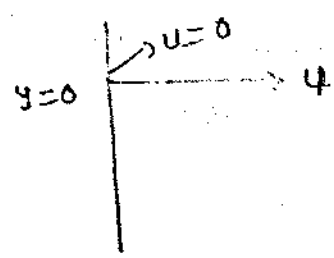
$$\mu_l \times \frac{\partial u}{\partial y}\, dx \times 1 = (\rho_l - \rho_v)(\delta - y)\, dx \times 1 \times g$$

$$\frac{\partial u}{\partial y} \cong \frac{du}{dy} = \frac{(\rho_l - \rho_v)(\delta - y) g}{\mu_l}$$

$$\frac{du}{dy} = \frac{g(\rho_l - \rho_v)}{\mu_l}(\delta - y) \longrightarrow (3)$$

Let $\delta \cong \delta(x)$ is the thickness of condensate layer at any position of "x"

Boundary Condition : at wall $u=0$ at $y=0$



$$\int du = \frac{g(\rho_l - \rho_v)}{\mu_l} \times \int (\delta - y)\, dy$$

$$u(y) = \frac{g(\rho_l - \rho_v)}{\mu_l}\left[\delta y - \frac{1}{2} y^2\right] \longrightarrow (4)$$

mass flow rate of condensate $m(x)$

$$m(x) = \int_0^\delta \rho_l \times u\, dy \times 1$$

$$m(x) = \int_0^\delta \rho_l \times \frac{g(\rho_l - \rho_v)}{\mu_l} \times \left[\delta y - \frac{1}{2} y^2\right] dy$$

$$m(x) = \frac{\rho_l g(\rho_l-\rho_v)}{\mu_l}\left[\frac{\delta y^2}{2} - \frac{1}{2}\frac{y^3}{3}\right]_0^{\delta}$$

$$= \frac{\rho_l g(\rho_l-\rho_v)}{\mu_l}\left[\frac{\delta^3}{2} - \frac{1}{2}\frac{\delta^3}{3}\right]$$

$$= \frac{\rho_l g(\rho_l-\rho_v)}{\mu_l}\left[\frac{3\delta^3-\delta^3}{6}\right] = \frac{\rho_l g(\rho_l-\rho_v)}{\mu_l} \times \frac{2\delta^3}{6}$$

$$m(x) = \frac{\rho_l g(\rho_l-\rho_v)\,\delta^3}{\mu_l \times 3} \longrightarrow (5)$$

differentiate equation (5) w.r.to "δ"

$$\frac{dm}{d\delta} = \frac{\rho_l g(\rho_l-\rho_v)\times 3\delta^2}{3\mu_l}$$

$$dm = \frac{\rho_l g(\rho_l-\rho_v)\,\delta^2}{\mu_l}\,d\delta \longrightarrow (6) \qquad d\delta = \frac{dm \times \mu_l}{\rho_l g(\rho_l-\rho_v)\delta^2}$$

rate of heat transfer with respect to the rate of condensation

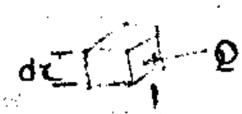
rate of heat transfer with respect to the rate of condensation

∴ latent heat is rejected from vapour to form condensate

$$dQ = h_{fg}\,dm \longrightarrow (7)$$

As per the assumption the heat transfer takes place at the condensate layer is pure conduction

$$dQ = k_l\frac{(T_{sat}-T_s)}{\delta} \times dx \times 1 \longrightarrow (8)$$



$$dm = \frac{dQ}{h_{fg}}$$

$$dx = \frac{dQ \times \delta}{k_l(T_{sat}-T_s)}$$

$$\frac{d\delta}{dx} = \frac{\left(\dfrac{dm \times \mu_l}{\rho_l g(\rho_l-\rho_v)\delta^2}\right)}{\dfrac{dQ \times \delta}{k_l(T_{sat}-T_s)}}$$

$$\frac{d\delta}{dx} = \frac{dm \times \mu_l}{\rho_l \times g(\rho_l - \rho_v)\delta^2} \times \frac{k_l (T_{sat} - T_s)}{h_{fg} \times dm \times \delta}$$

$$\frac{d\delta}{dx} = \frac{\mu_l k_l (T_{sat} - T_s)}{\rho_l g (\rho_l - \rho_v) h_{fg} \delta^3} \longrightarrow (9)$$

Integrating the equation (9) & Applying boundary conditi

$\delta = 0$ at $x = 0$

$x = 0 \Rightarrow \delta = 0$

$$d\delta = \frac{\mu_l k_l (T_{sat} - T_s)}{g \times \rho_l (\rho_l - \rho_v) \times h_{fg}} \times \frac{1}{\delta^3} dx$$

$$\int \delta^3 \, d\delta = \frac{\mu_l k_l (T_{sat} - T_s)}{g \times \rho_l (\rho_l - \rho_v) h_{fg}} \int dx$$

$$\frac{\delta^4}{4} = \frac{\mu_l k_l (T_{sat} - T_s)}{g \rho_l (\rho_l - \rho_v) h_{fg}} \times x$$

$$\delta^4 = \frac{4 \mu_l k_l (T_{sat} - T_s)}{g \rho_l (\rho_l - \rho_v) h_{fg}} x$$

$$\delta = \left[\frac{4 \mu_l k_l (T_{sat} - T_s)}{g \rho_l (\rho_l - \rho_v) h_{fg}} x \right]^{1/4} = \left[\frac{4 \mu_l k_l \Delta T x}{g \rho_l (\rho_l - \rho_v) h_{fg}} \right]$$

According to the Nusselt theory

$$dx \times 1 \times h(x) (T_{sat} - T_s) = k_l \frac{(T_{sat} - T_s)}{\delta x} \times dx \times 1$$

$$h(x) (T_{sat} - T_s) = k_l \frac{(T_{sat} - T_s)}{\delta x}$$

$$h_x = \frac{k_l}{\delta(x)} \rightarrow \text{local heat transfer coefficient}$$

$$h_x = \frac{k_l}{\left[\dfrac{4\mu_l k_l (T_{sat} - T_s)x}{g \rho_l (\rho_l - \rho_v) h_{fg}}\right]^{1/4}}$$

$$h_x = \frac{\left[g \times \rho_l \times (\rho_l - \rho_v) h_{fg}\right]^{1/4} \times k_l'}{\left[4 \mu_l (T_{sat} - T_s) x\right]^{1/4} \cdot k_l^{1/4}}$$

$$h_x = \left(\frac{g \times \rho_l (\rho_l - \rho_v) h_{fg} \; k_l^3}{4 \mu_l (T_{sat} - T_s) x}\right)^{1/4}$$

Averag heat transfer coefficient $\quad h_m = \frac{1}{L}\int_0^L h_x \, dx$

$$h_m = \frac{4}{3} h_x \Big|_{x=L}$$

$$h_m = 0.943 \left[\frac{g \rho_l (\rho_l - \rho_v) h_{fg} k_l^3}{\mu_l (T_{sat} - T_s) \times L}\right]^{1/4}$$

Inclined surface

$$h_m = 0.943 \left(\frac{g \rho_l (\rho_l - \rho_v) h_{fg} k_l^3 \times \sin\theta}{\mu_l (T_{sat} - T_s) \times L}\right)^{1/4}$$

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